Graphs Are Everywhere

Christopher Hoult
@choult

Who is this guy?

Practicing PHP commercially since 2004

Senior Software Engineer at Datto

Formerly at DataSift and Time Out Digital

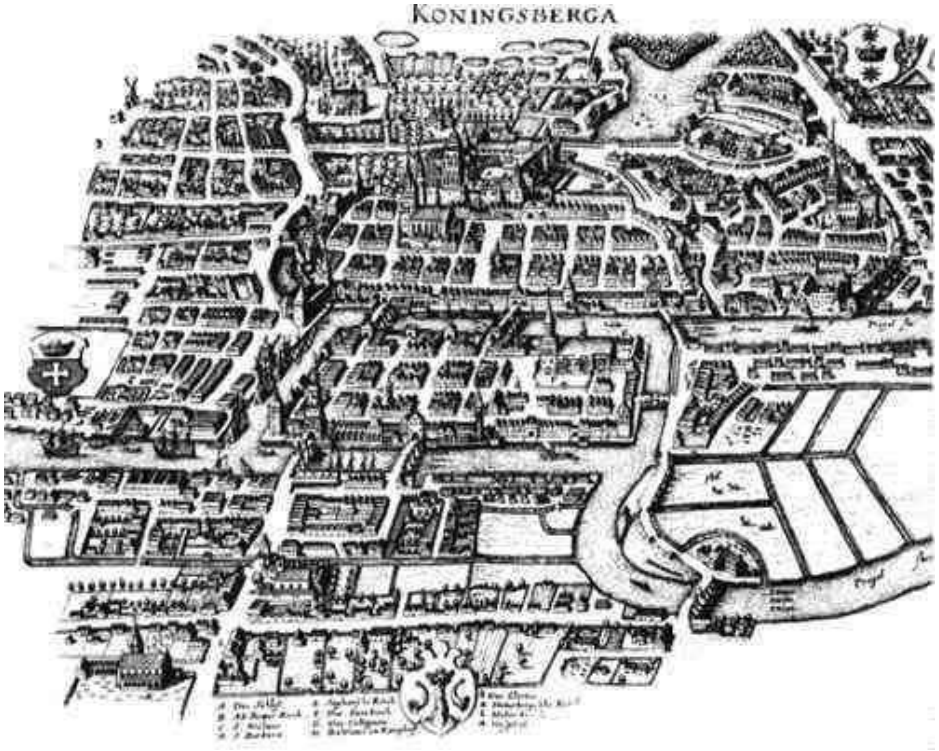
Founder/co-organizer of the PHP Berkshire User Group

I also act

Graphs are everywhere

The Seven Bridges of Königsberg

Leonhard Euler, Prussia,
1736



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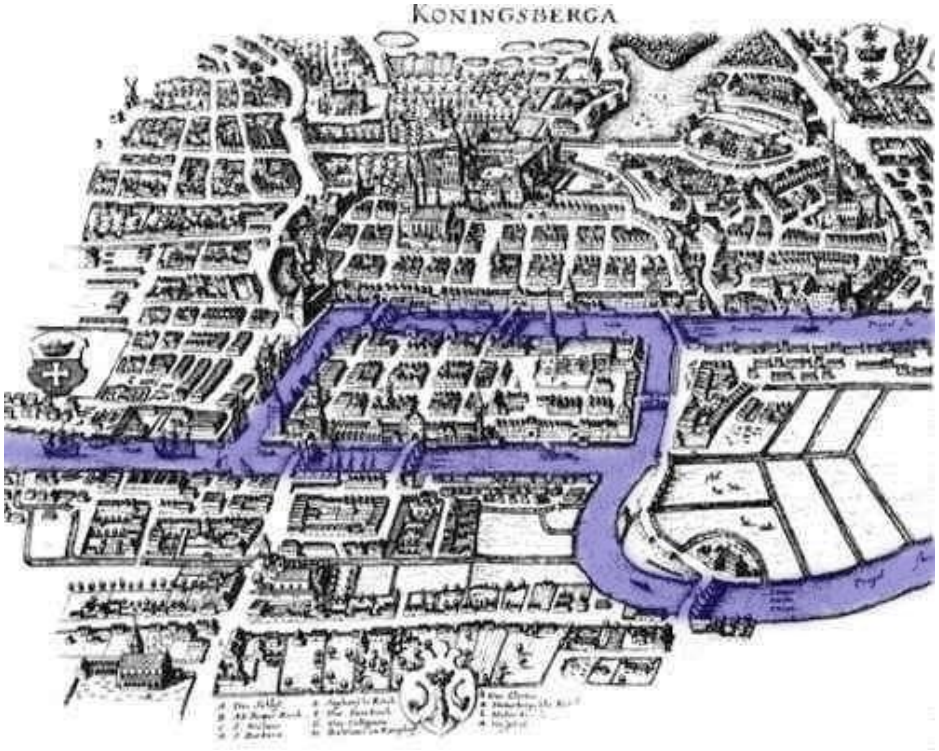

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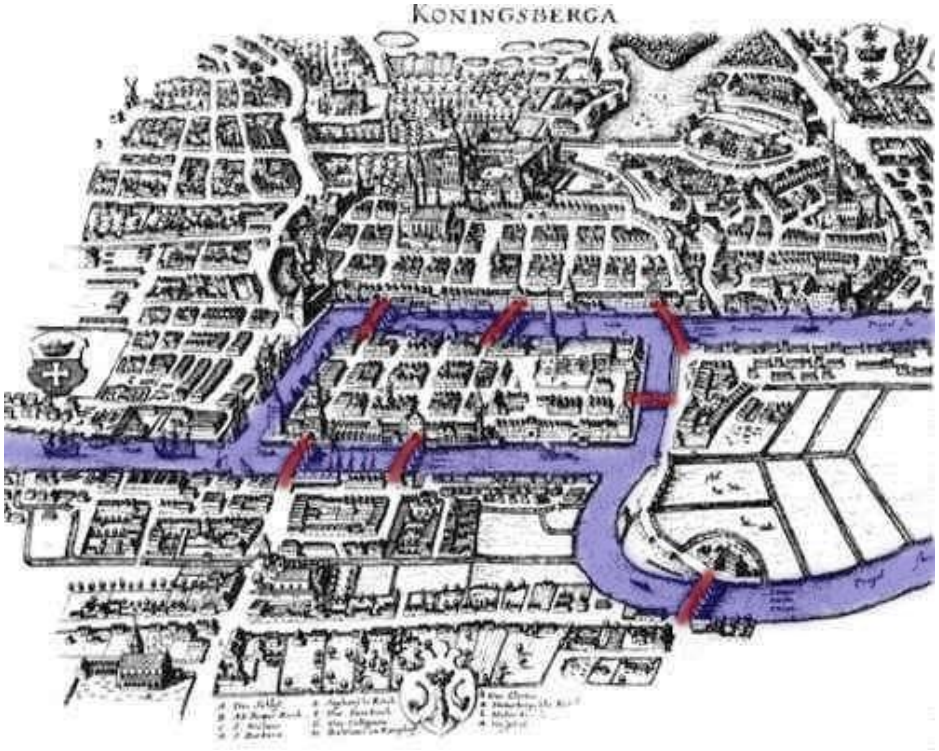
“Can you walk through the city, crossing each bridge only once?”



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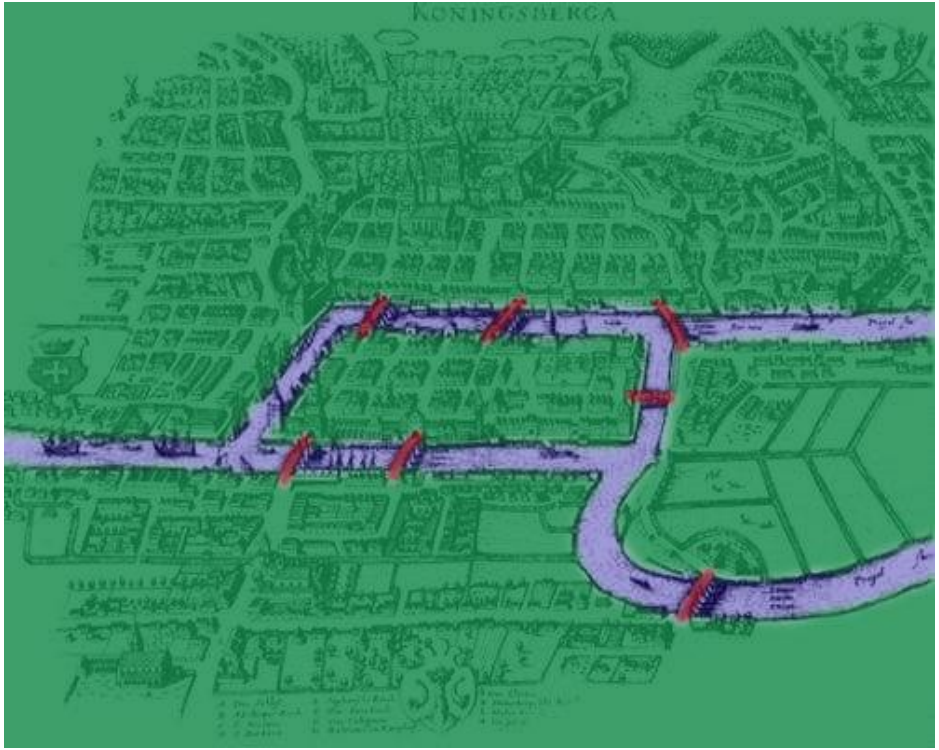
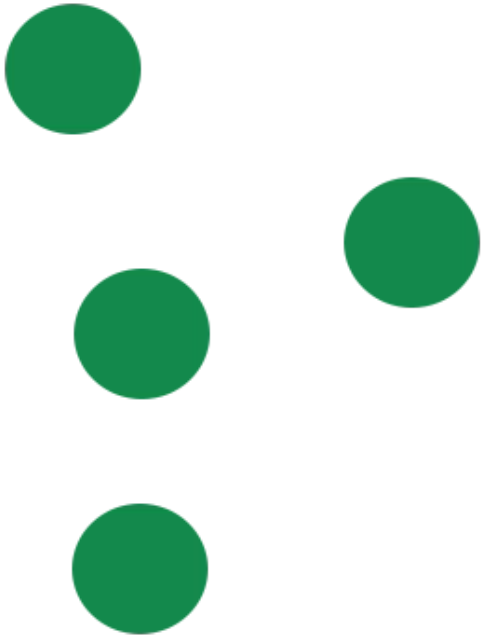
Reduce to four land-masses, and seven connections



The Seven Bridges of Königsberg

Nodes (or vertices)



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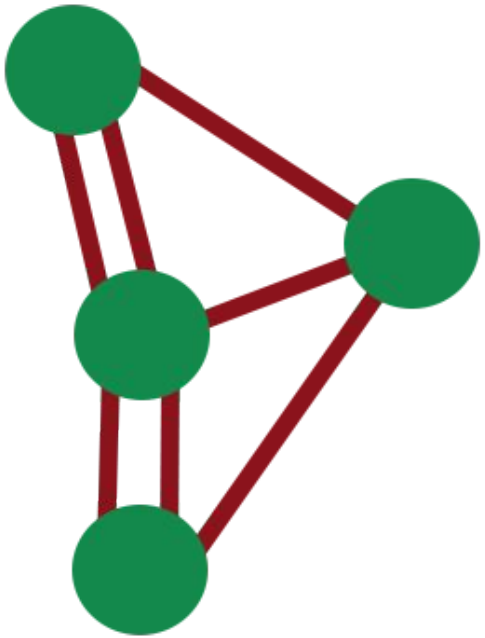

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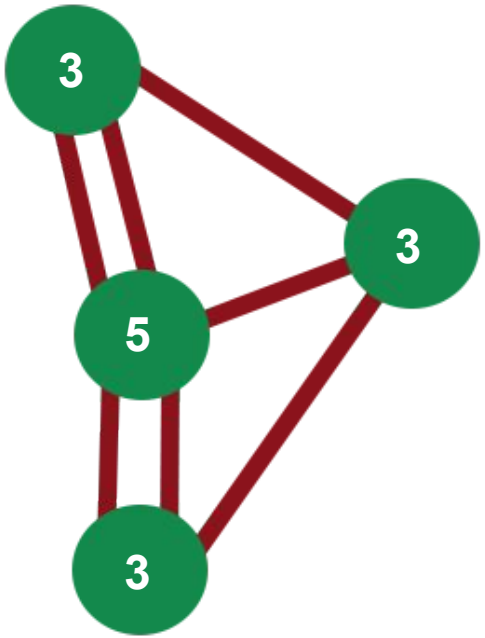

The Seven Bridges of Königsberg

Nodes (or vertices)

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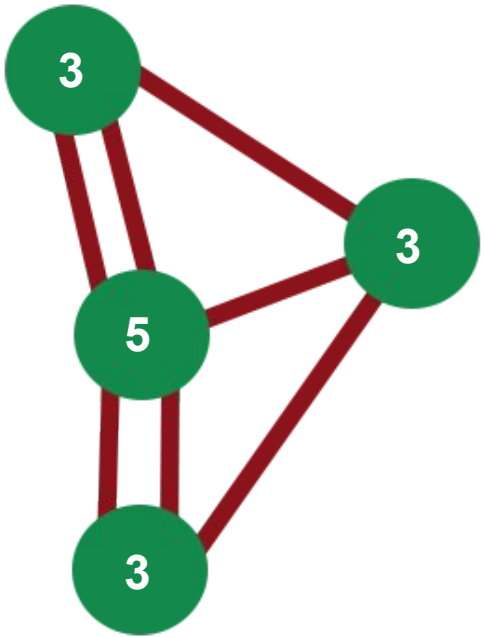
Example of an undirected graph - you can go either way over each bridge.



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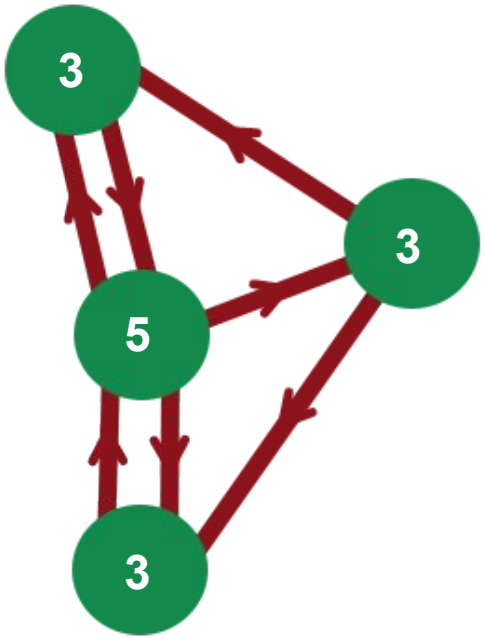
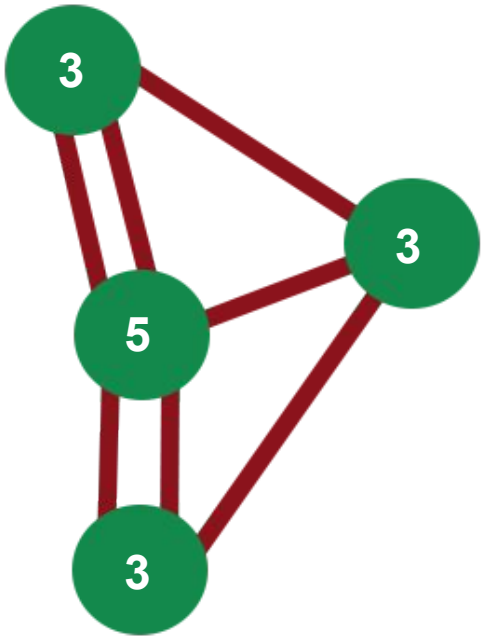
Example of an undirected graph - you can go either way over each bridge.

A one-way system would be a directed graph.



The Seven Bridges of Königsberg

To get into and out of a node, there must be an even number of edges; its *degree* must be even.



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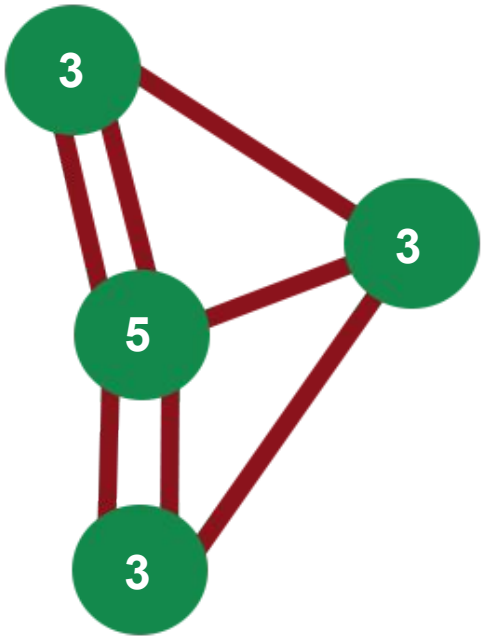

The Seven Bridges of Königsberg

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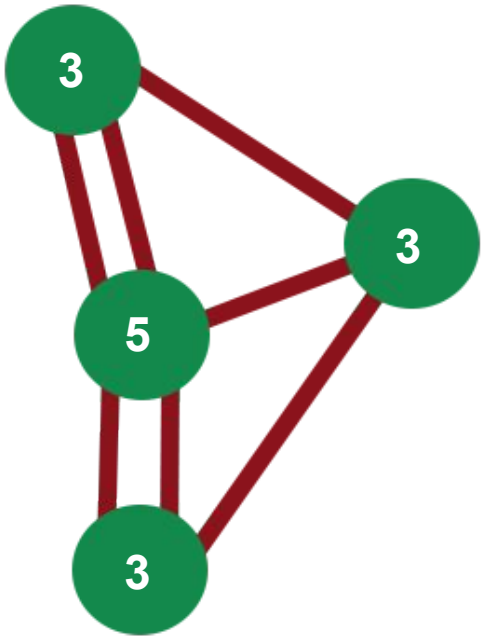
Therefore, only possible if zero or two nodes of odd degree - these will be start and end.



Pathfinding

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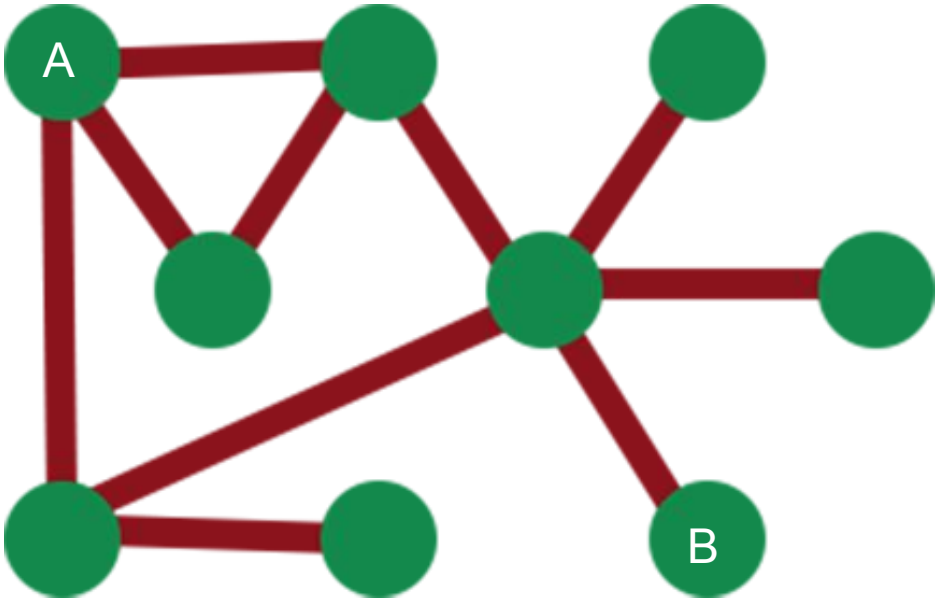
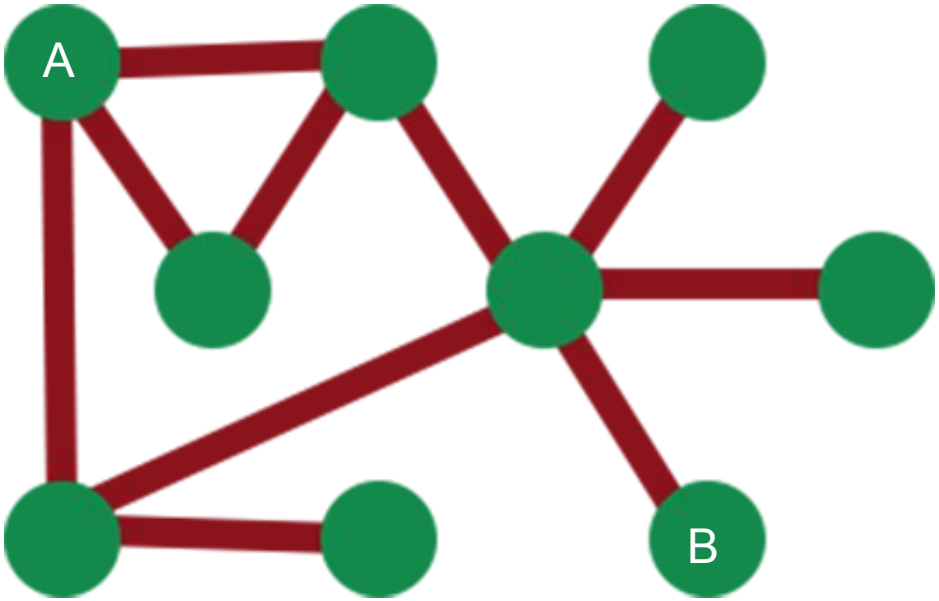
Problem: Find your way from arbitrary node A to arbitrary node B.



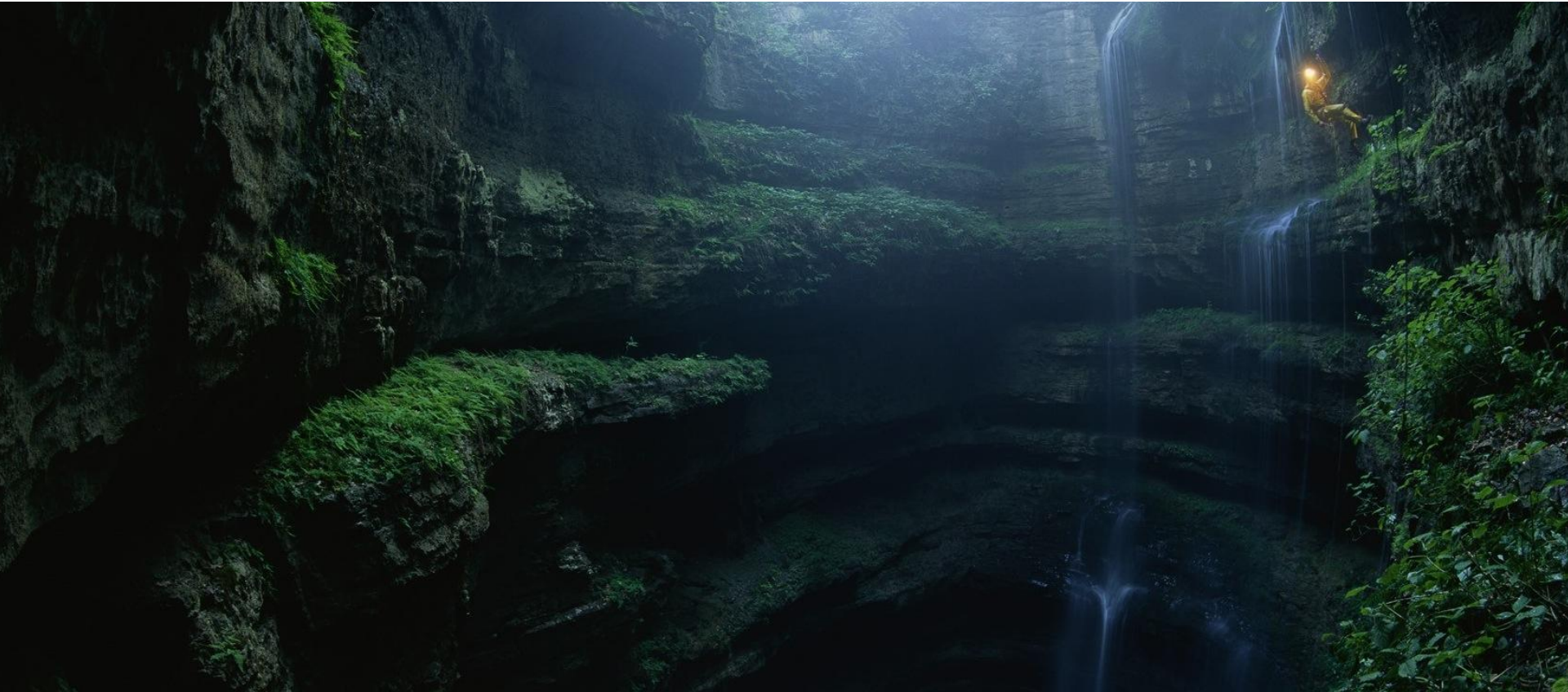
Pathfinding

Problem: Find your way from arbitrary node A to arbitrary node B.

Two approaches: depth-first and breadth-first.



Pathfinding: Depth-first



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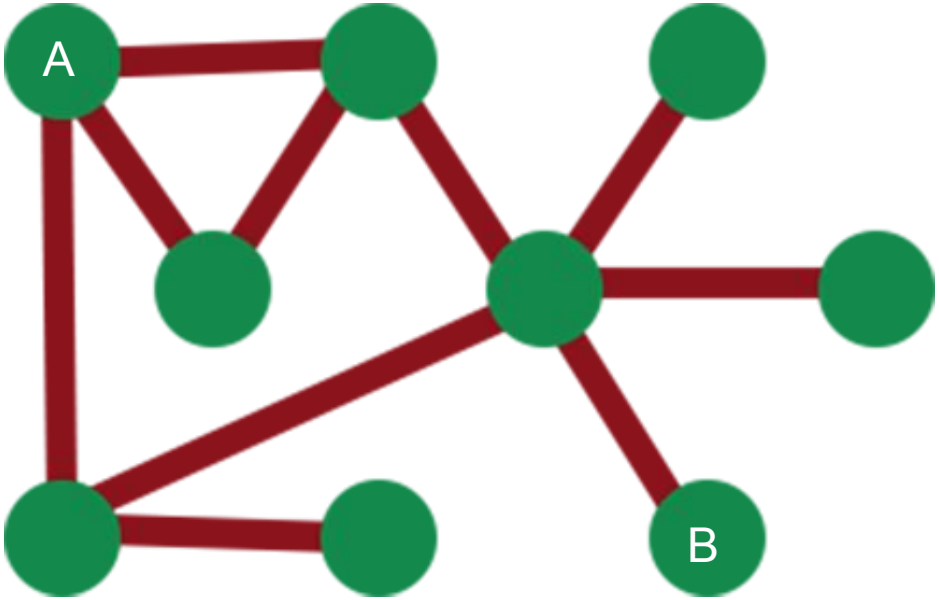

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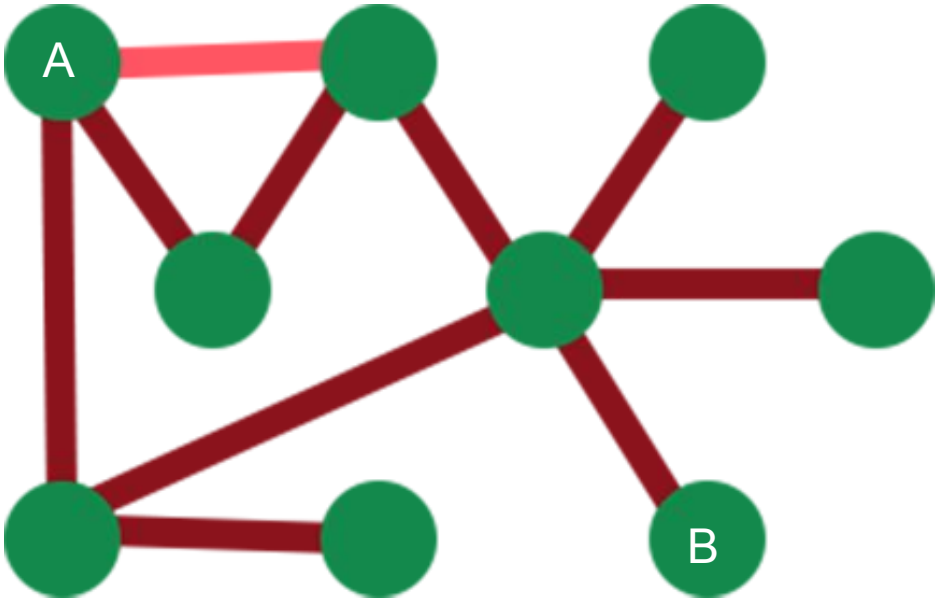

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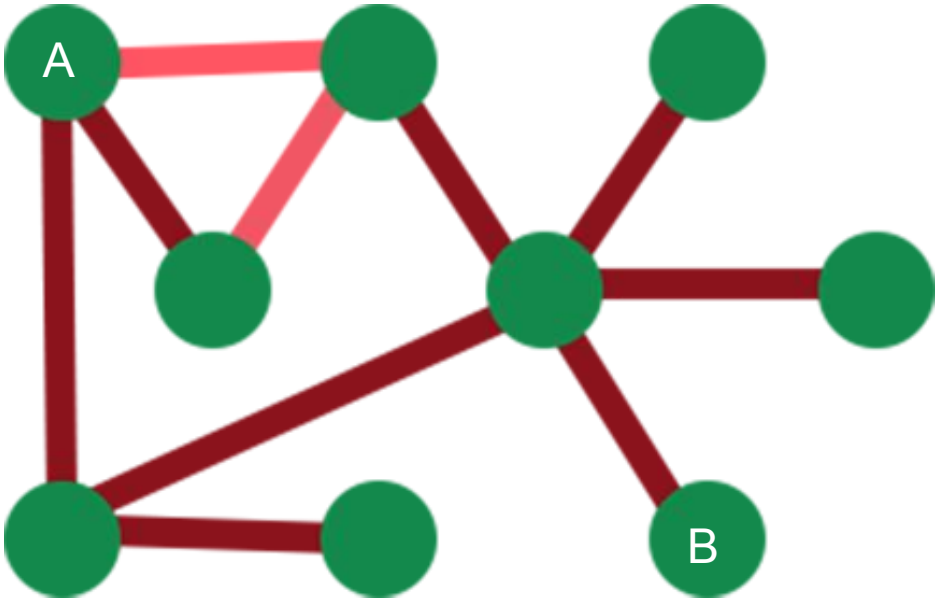

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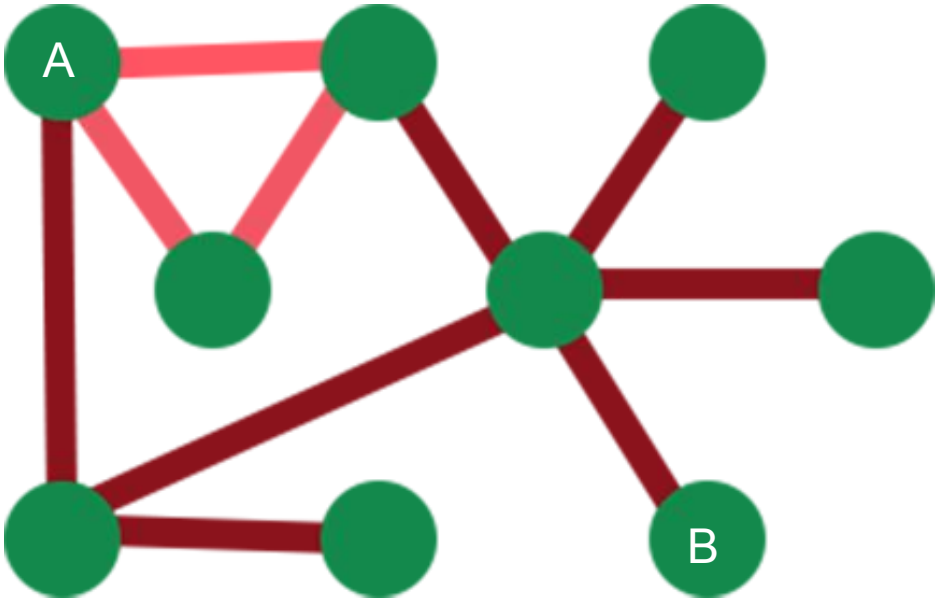
A cycle is where a collection of nodes are interconnected such that a path will loop back on itself.



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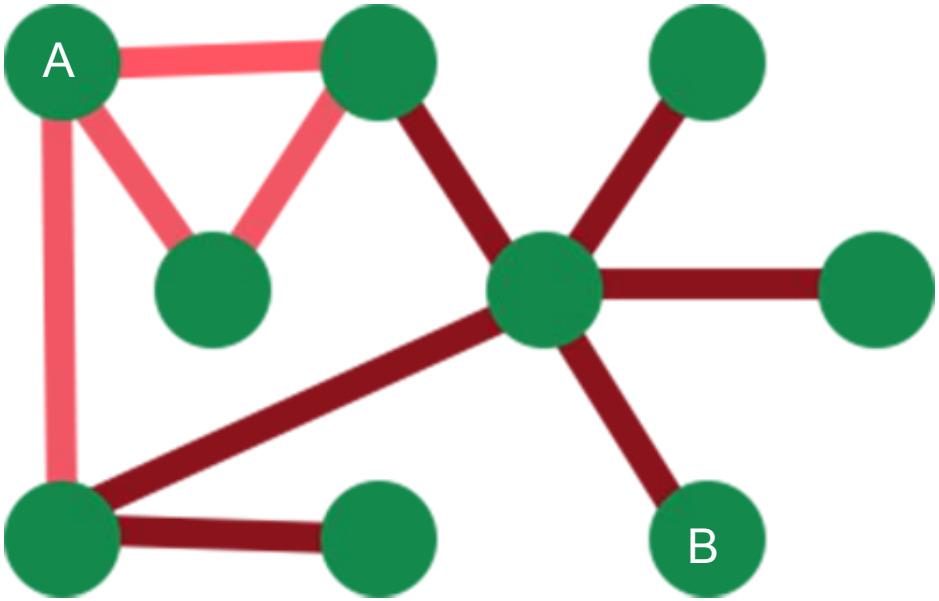

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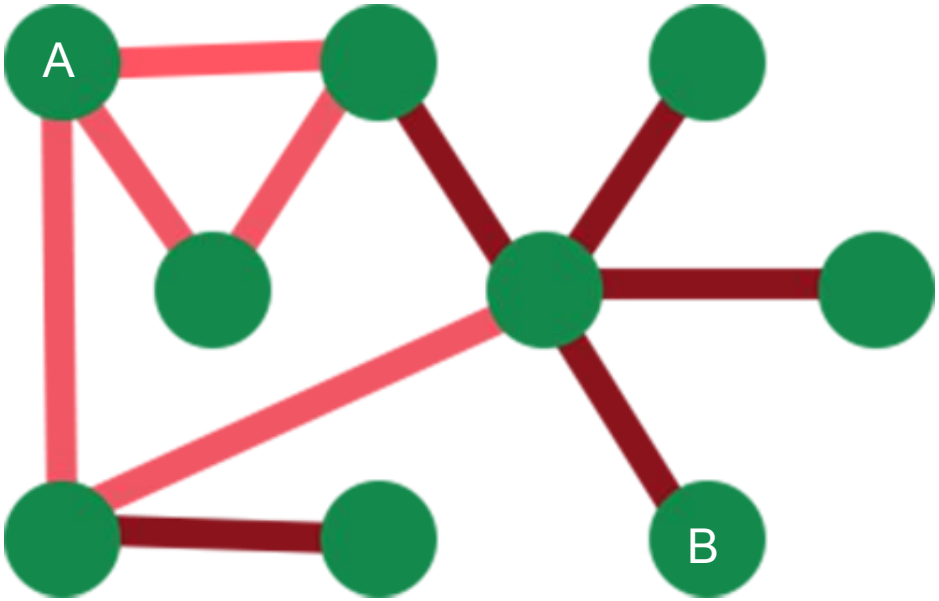

Pathfinding: Depth-first

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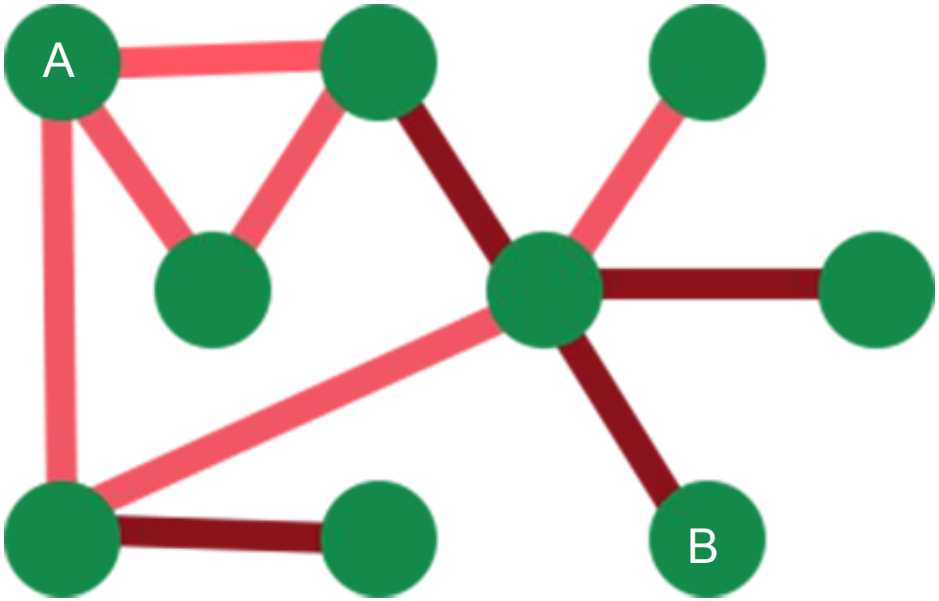
We've now hit a dead-end, and can go no further - so backtrack and try again.



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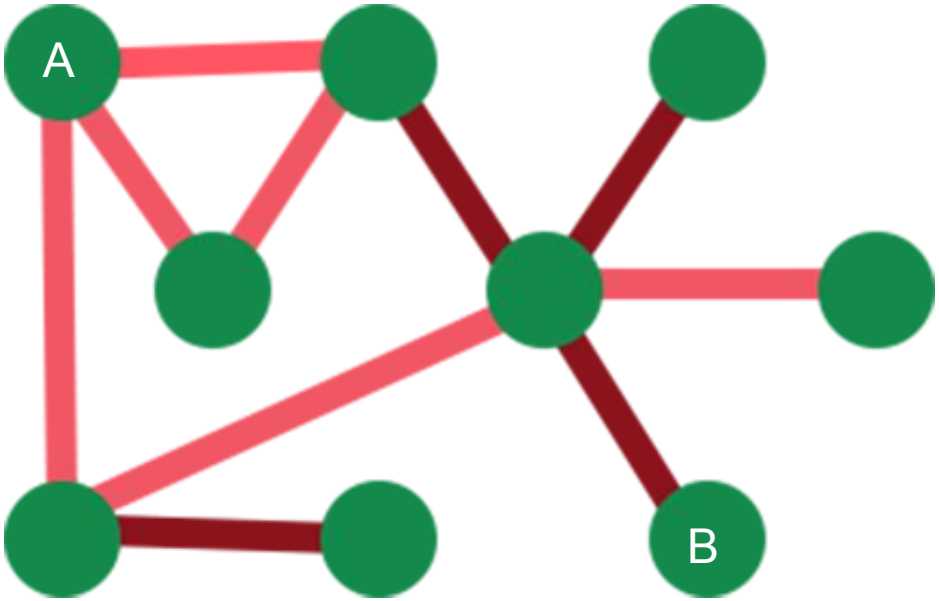
Best-case: we get to node B as quickly as possible.

Worst-case: we get to node B last, having explored the whole graph!



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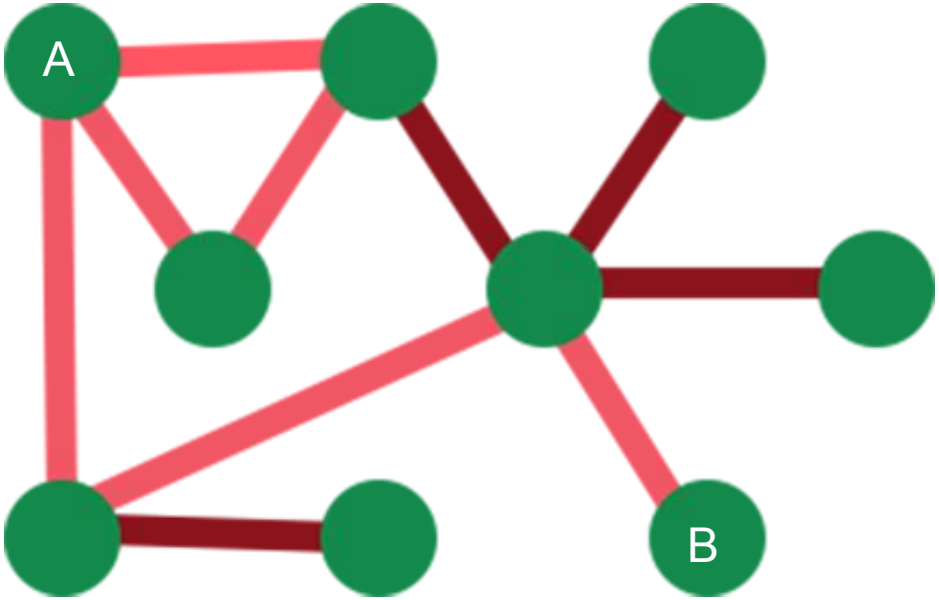

Depth-first: Modelling language

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Sentences can be modelled as directed graphs:

The **cat** **sat** **on** **the** **mat**

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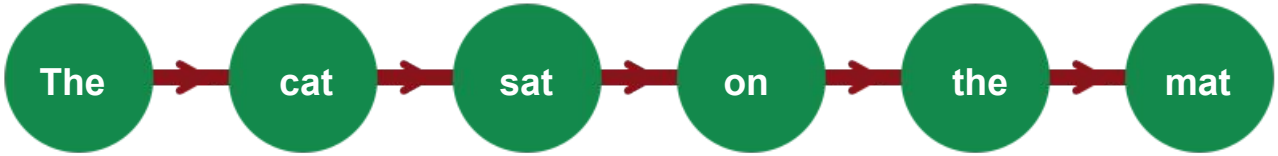
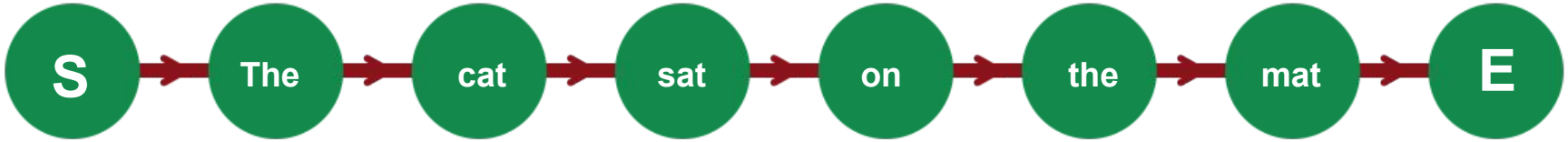


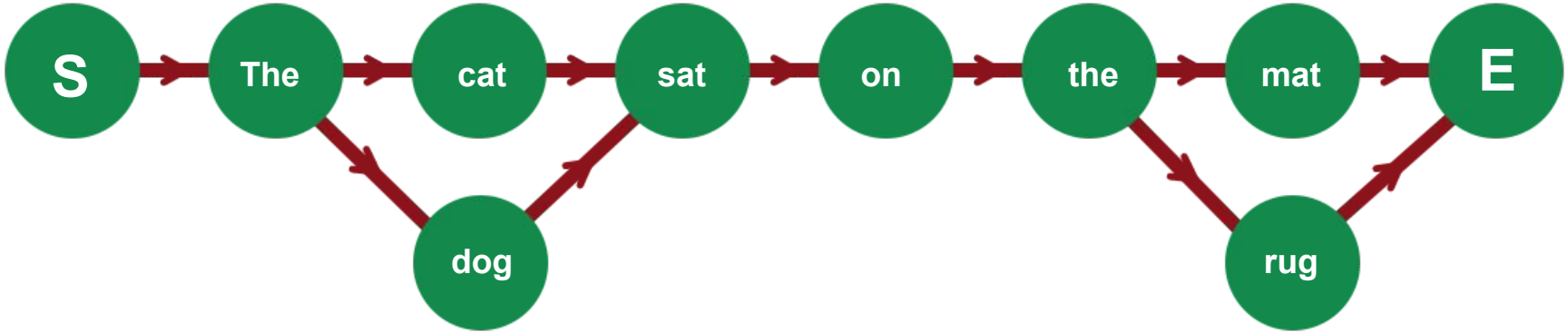
Depth-first: Modelling languages

We can then add special start and end nodes:



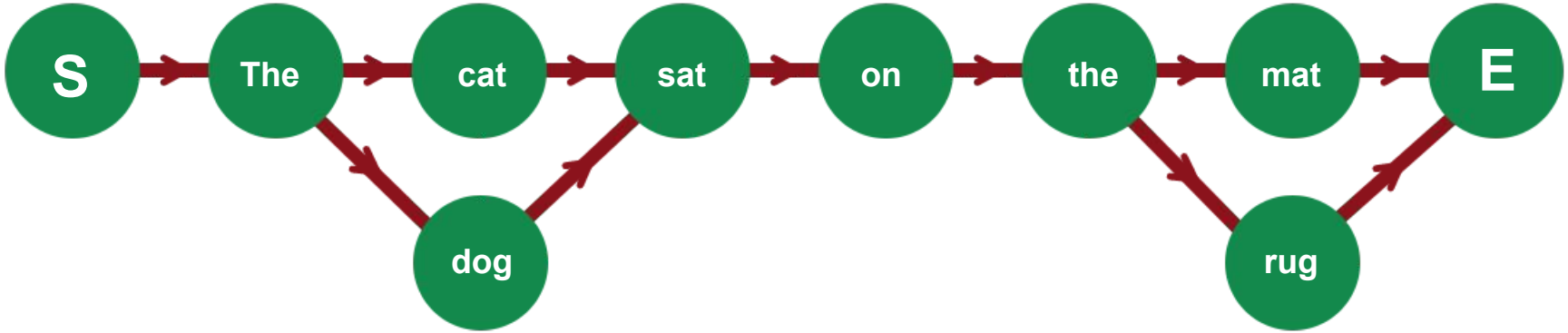
Depth-first: Modelling languages

Now add another sentence:



Depth-first: Modelling languages

We can now generate a novel sentence by randomly traversing it:



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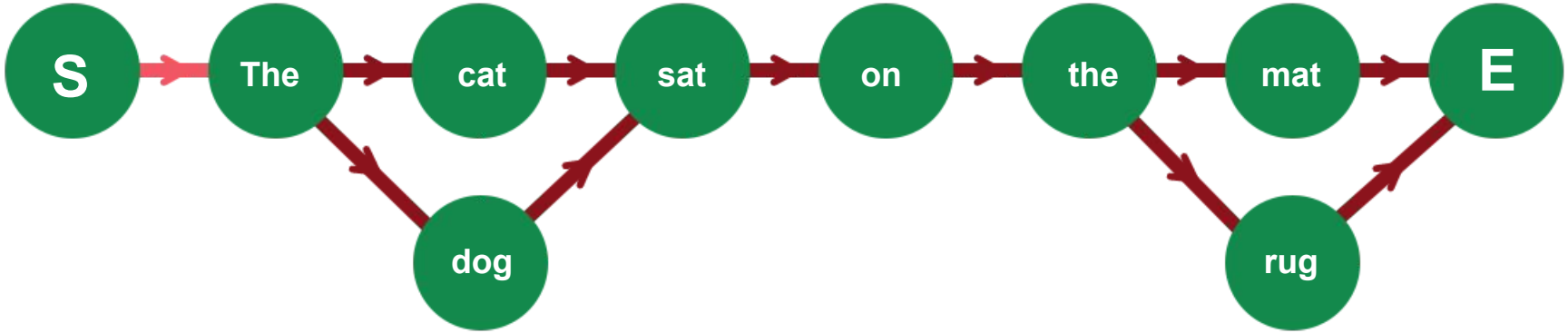

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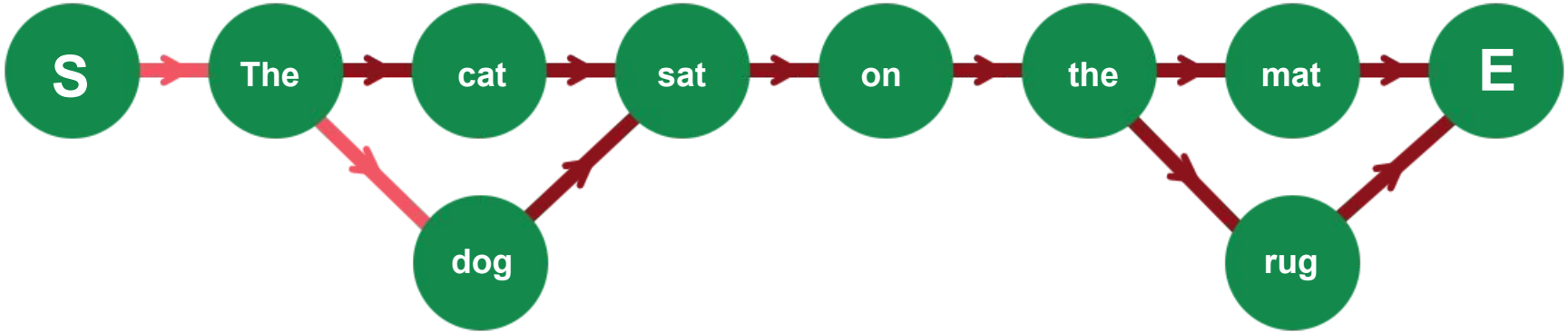

The dog

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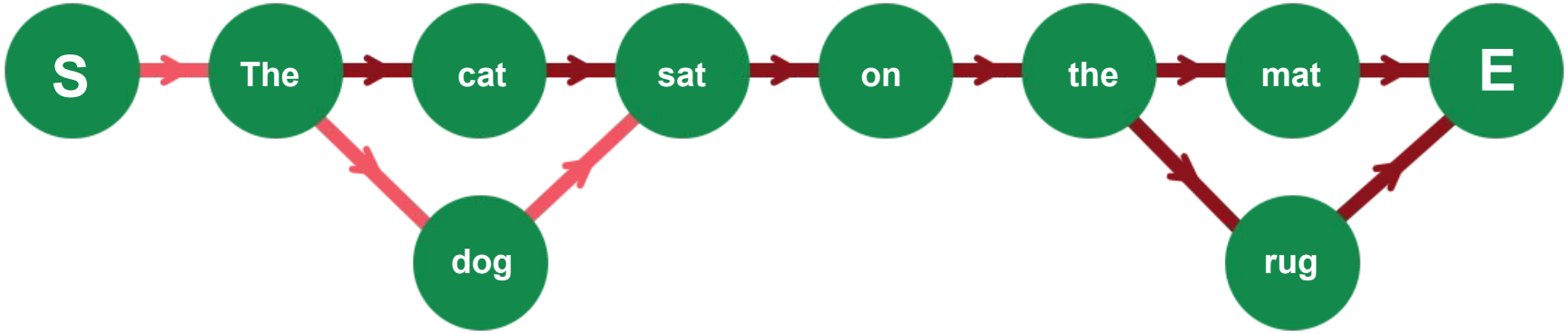

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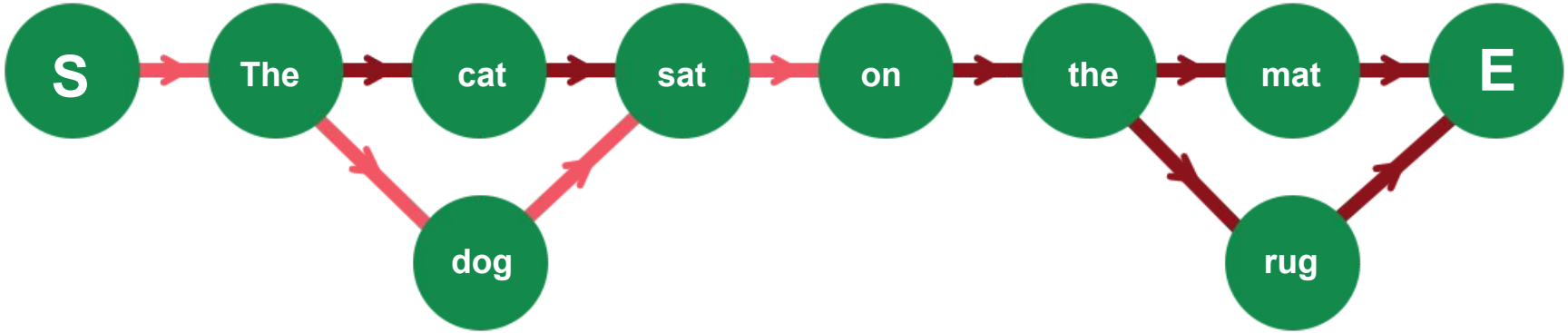

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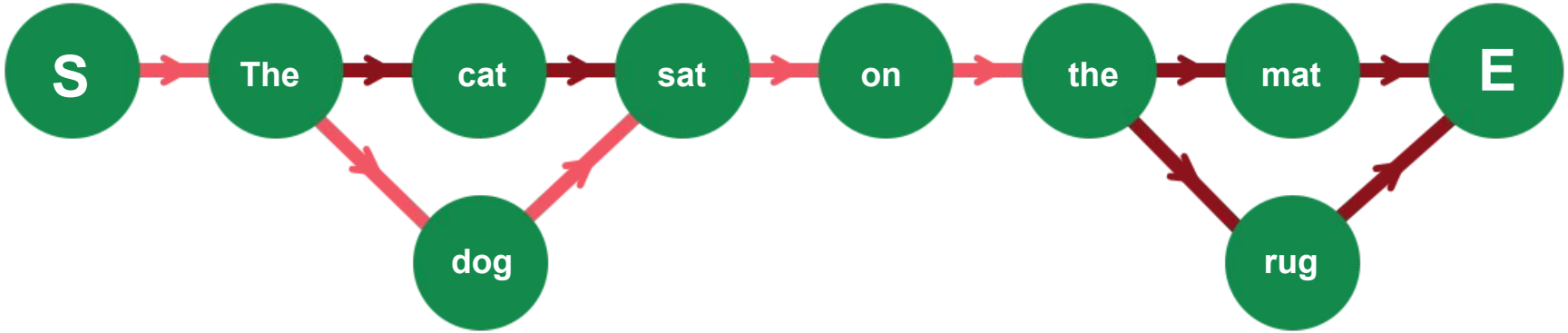

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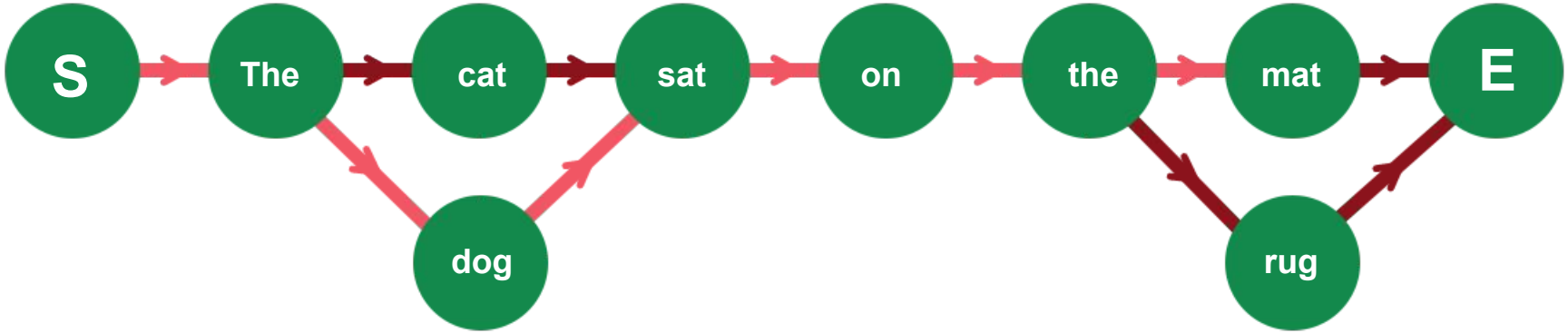
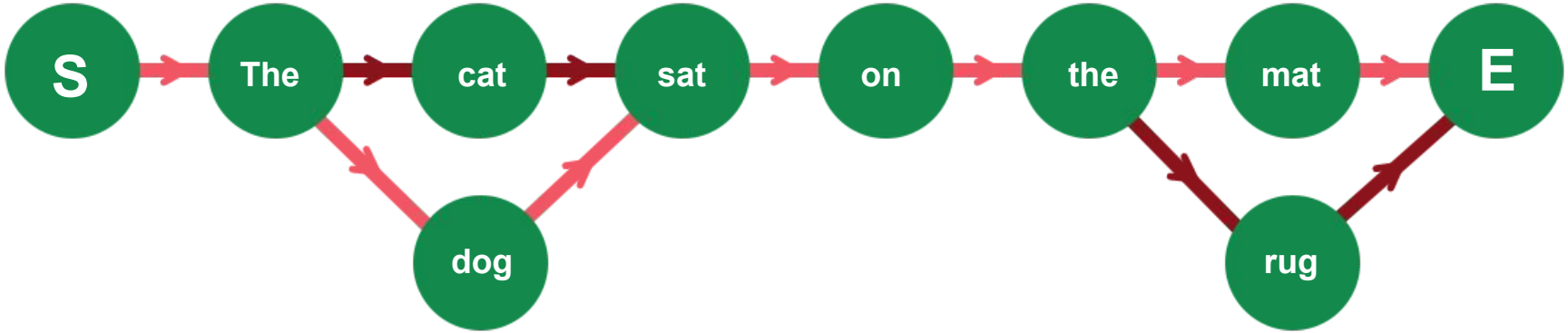


The dog sat on the mat

Depth-first: Modelling languages

When we find the end node, we have a (structurally sound) sentence.



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Depth-first: Modelling sentences

Daily Mail Headline Generator

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http://choult.com/dm

Source: http://bit.ly/2aNrWF4

Depth-first: Modelling sentences

NORTH KOREA FIRES A 60FT WELL IN THE FIRST JOB - SAYS HE WAS SHOT DEAD HIS EMOTIONAL VISIT TO LONG-FORGOTTEN 1846 WILD WEST SETTLERS WHO WON BY ISIS ATTACK

Depth-first: Modelling sentences

MICHAEL PHELPS GETS HIS FRIEND'S JAPANESE AKITA DOG

Depth-first: Modelling sentences

AWE-INSPIRING MOMENT

AMERICA'S GOT TALENT ACT GOES

GLUTEN-FREE WITH HER BROTHER

Depth-first: Modelling sentences

AT5 Headline Generator

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http://choult.com/at5

Source: http://bit.ly/2aNrWF4

Depth-first: Modelling sentences

VROUW RIJDT MET SCOOTER WATER REGULIERSGRACHT IN JONGERENRECHTBANK: 'LAAT JE FOUT DEED'

Depth-first: Modelling sentences

VROUW RIJDT MET DRUGSGELD

Depth-first: Modelling sentences

TAXIBEDRIJF START MET OVERGEWICHT IN NIEUW-WEST GEOPEND: 'WONEN OP STAND'

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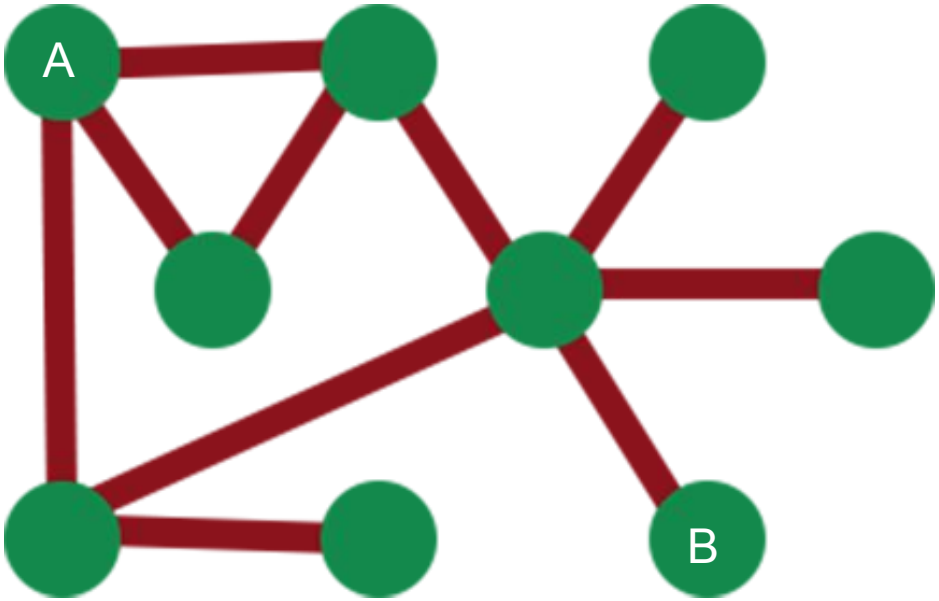
Problem: Find your way from arbitrary node A to arbitrary node B.

Breadth-first: check each node at depth 1, depth 2 etc. until node B is found.



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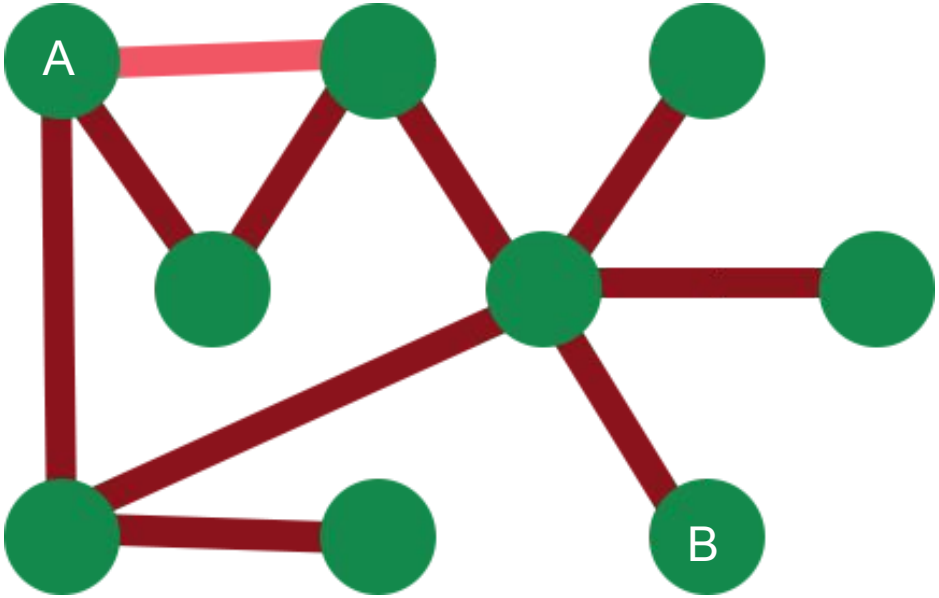

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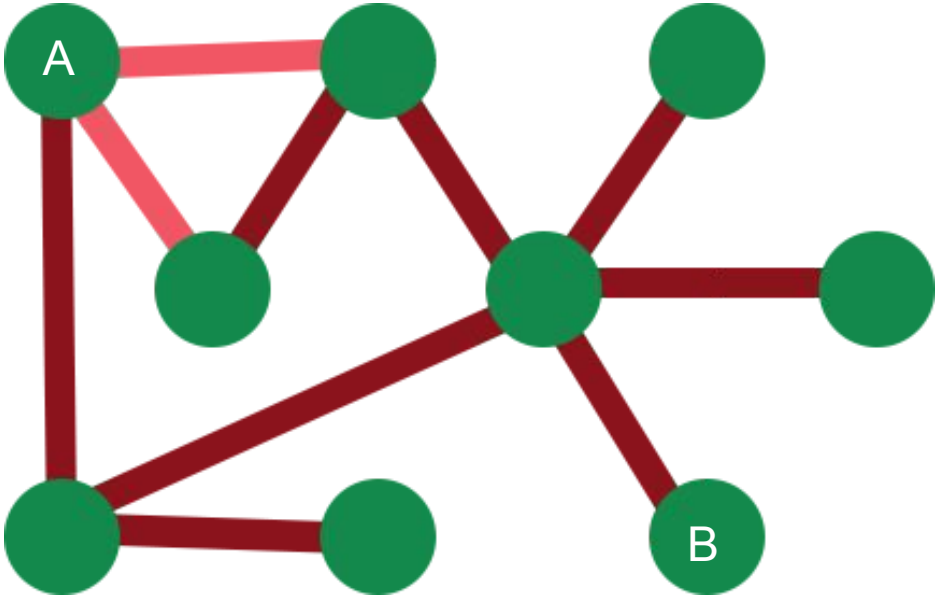

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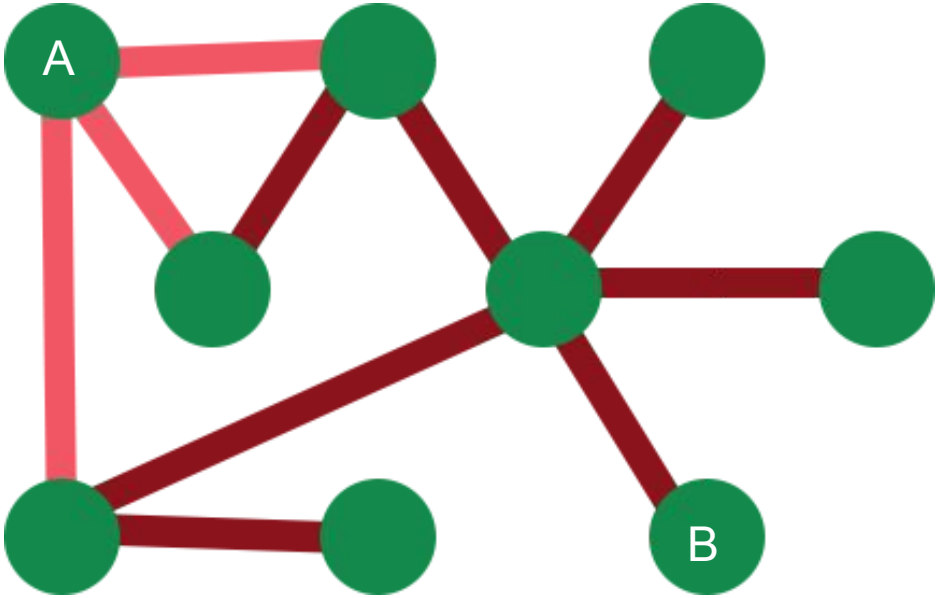

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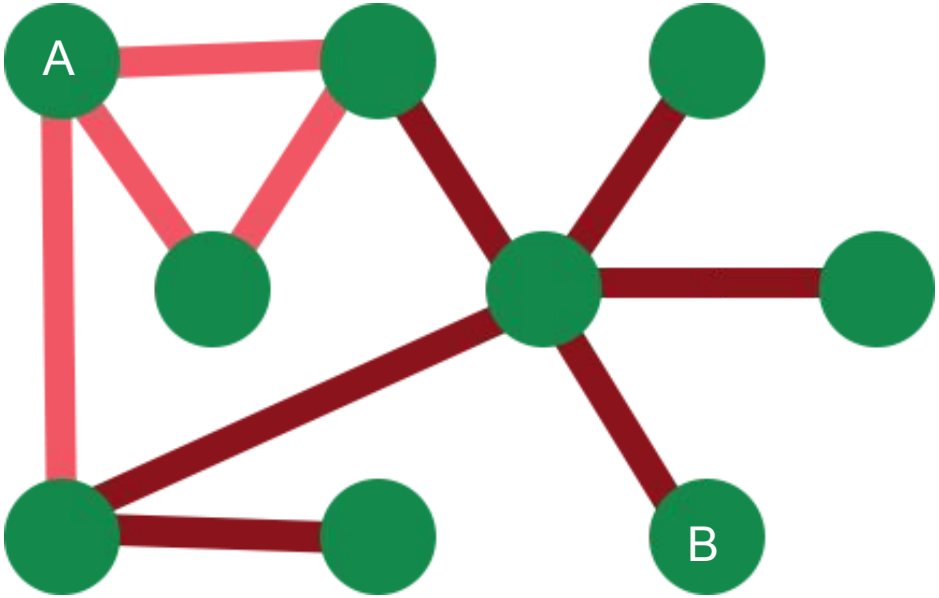

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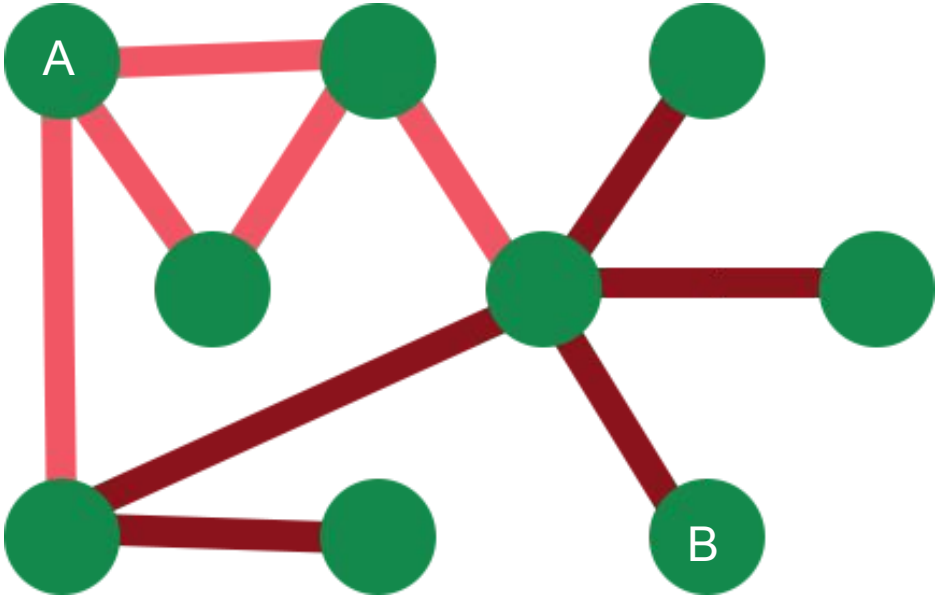

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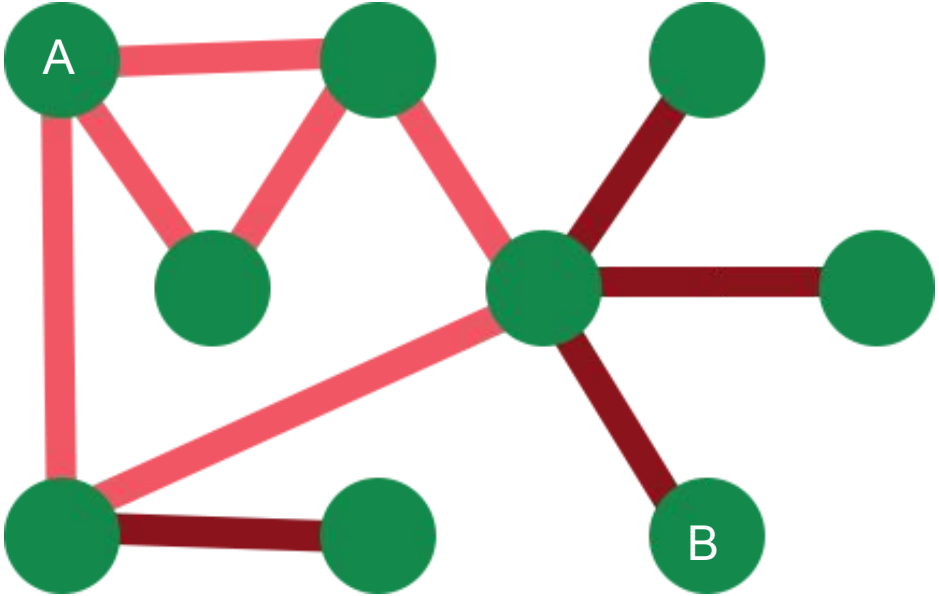

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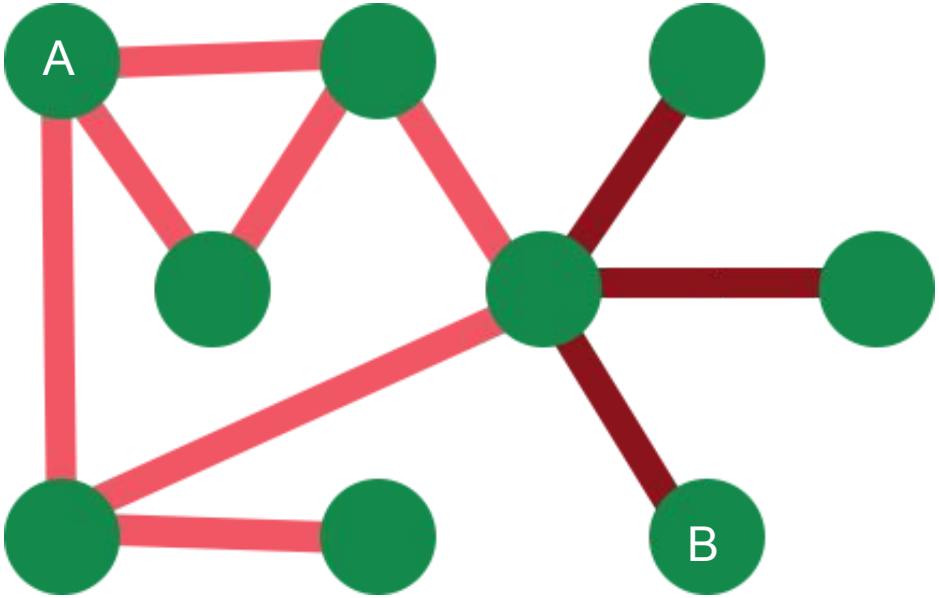

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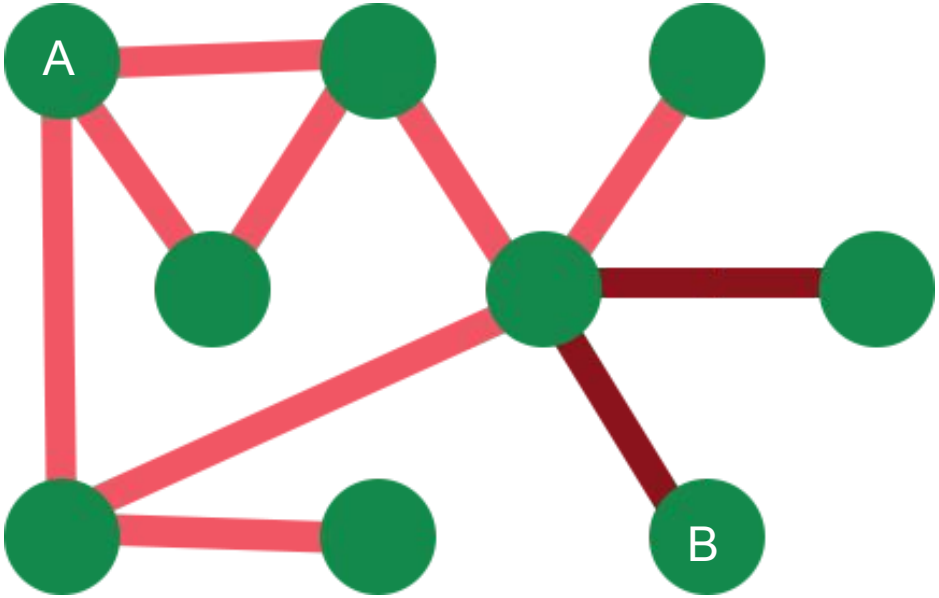

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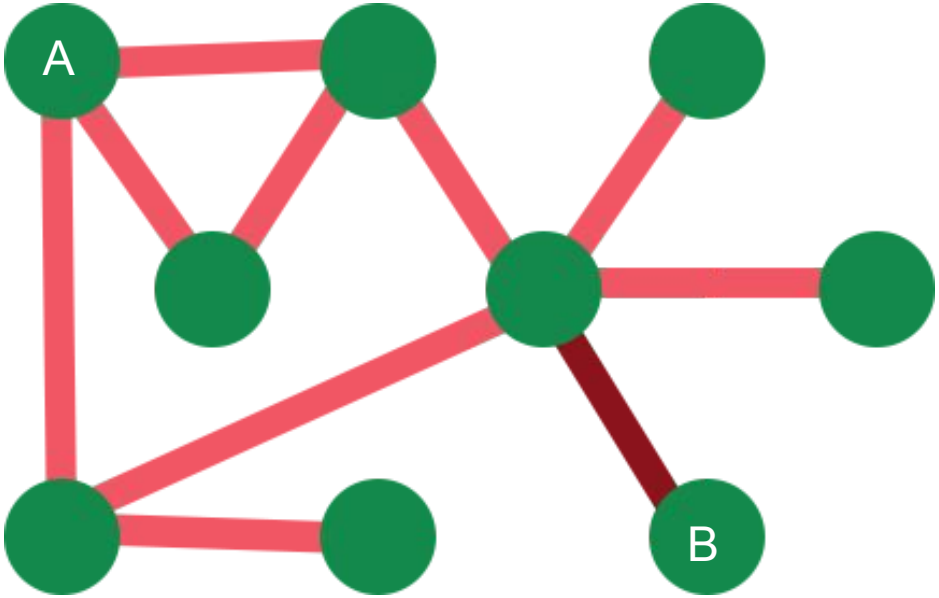
This can be slower, but will eventually find the shortest path.

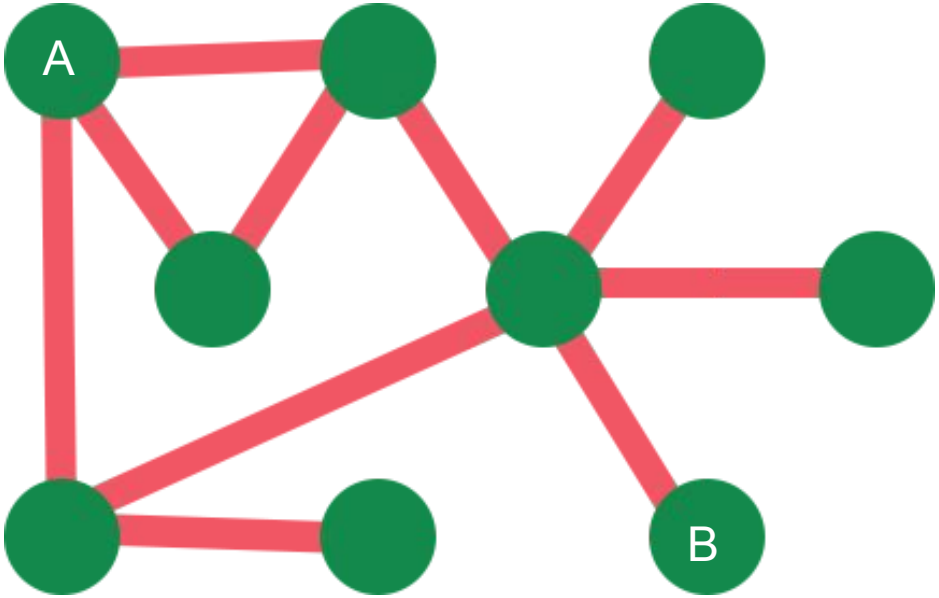


Pathfinding: Breadth-first

Problem: Find your way from arbitrary node A to arbitrary node B.

Breadth-first: check each node at depth 1, depth 2 etc. until node B is found.

This can be slower, but will eventually find the shortest path.



Breadth-first: The six degrees of Kevin Bacon

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An actor’s Bacon Number is the number of steps between them and Kevin Bacon via working with other actors on movies.

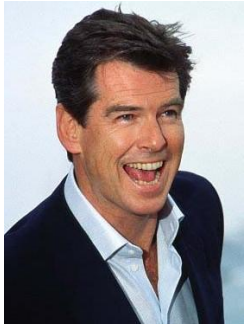
Breadth-first: The six degrees of Kevin Bacon

An actor's Bacon Number is the number of movies between them and Kevin Bacon via working with other actors.

Eg. Actor A worked with Actor B on movie 1; Actor B worked with Actor C on movie 2; Actor C worked with Kevin Bacon on movie 3 therefore Actor A's *Bacon Number* is 3

Breadth-first: The six degrees of Kevin Bacon

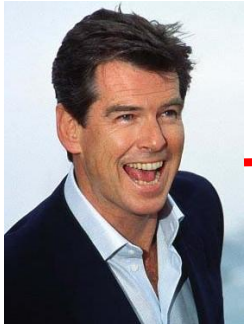
Pierce Brosnan has a Bacon Number of 2



Pierce Brosnan

Breadth-first: The six degrees of Kevin Bacon

Pierce Brosnan has a Bacon Number of 2



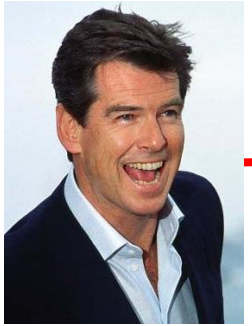
Pierce Brosnan

Was in

After the Sunset

Breadth-first: The six degrees of Kevin Bacon

Pierce Brosnan has a Bacon Number of 2



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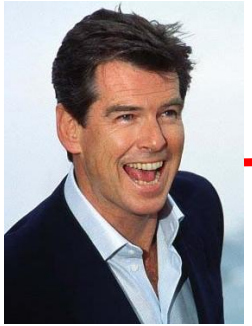

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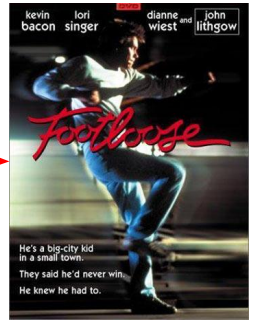

Footloose

Breadth-first: The six degrees of Kevin Bacon

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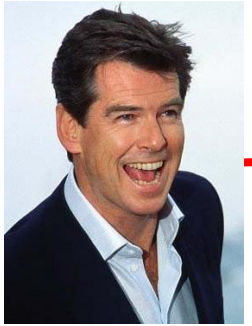

Pierce Brosnan

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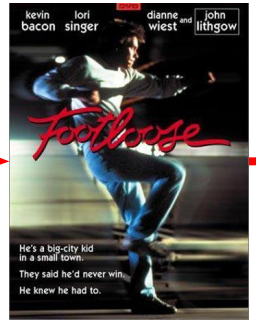

Footloose

With

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Breadth-first: The six degrees of Kevin Bacon

A graph to represent the search-space of IMDB (say) would be huge: 3m+ actors; 2m+ movies and TV shows.

Breadth-first: The six degrees of Kevin Bacon

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Depth-first search would be more susceptible to its worst-case scenario - it would just get lost without supreme luck!

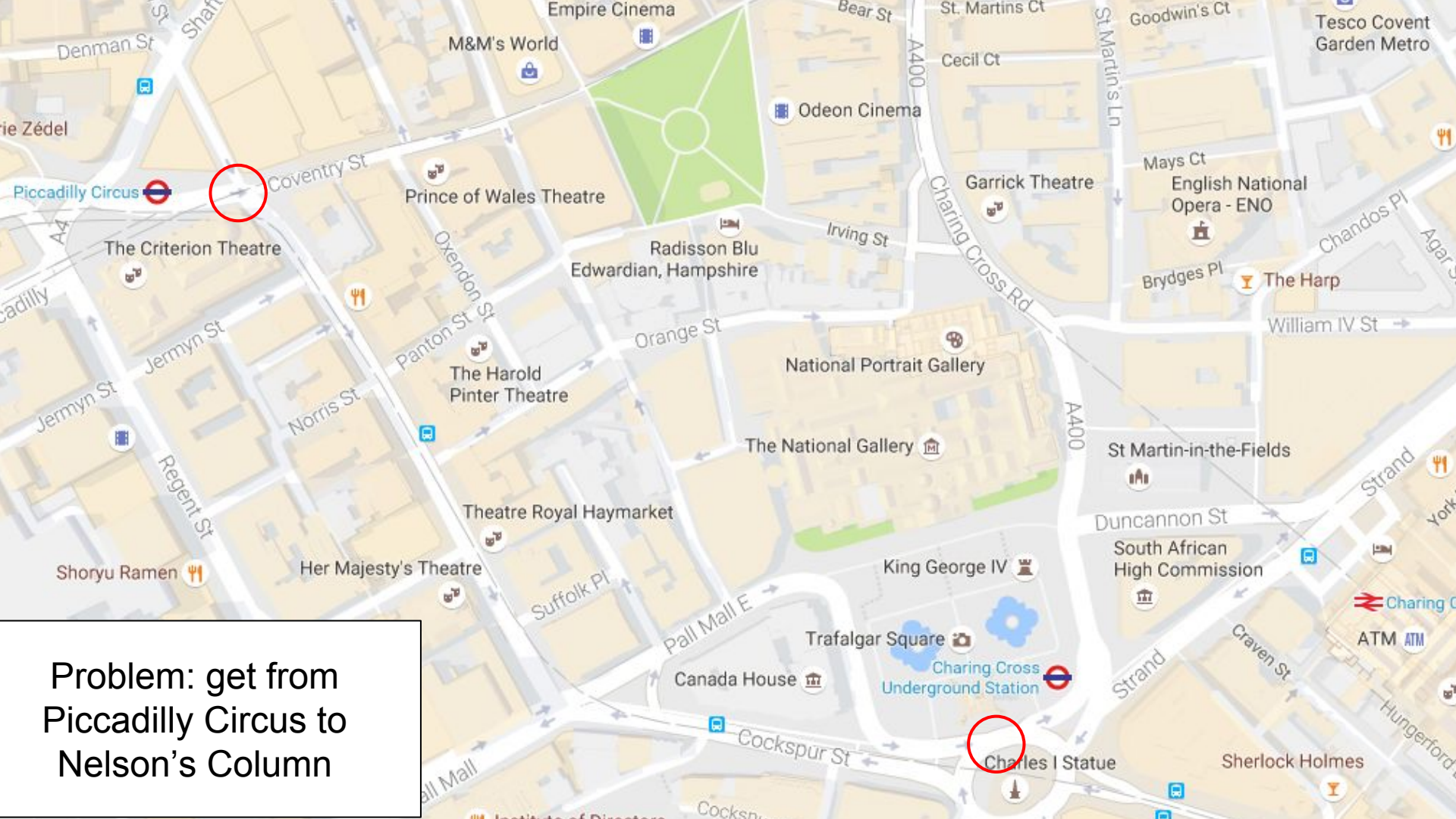
Breadth-first: The six degrees of Kevin Bacon

A graph to represent the search-space of IMDB (say) would be huge: 3m+ actors; 2m+ movies and TV shows.

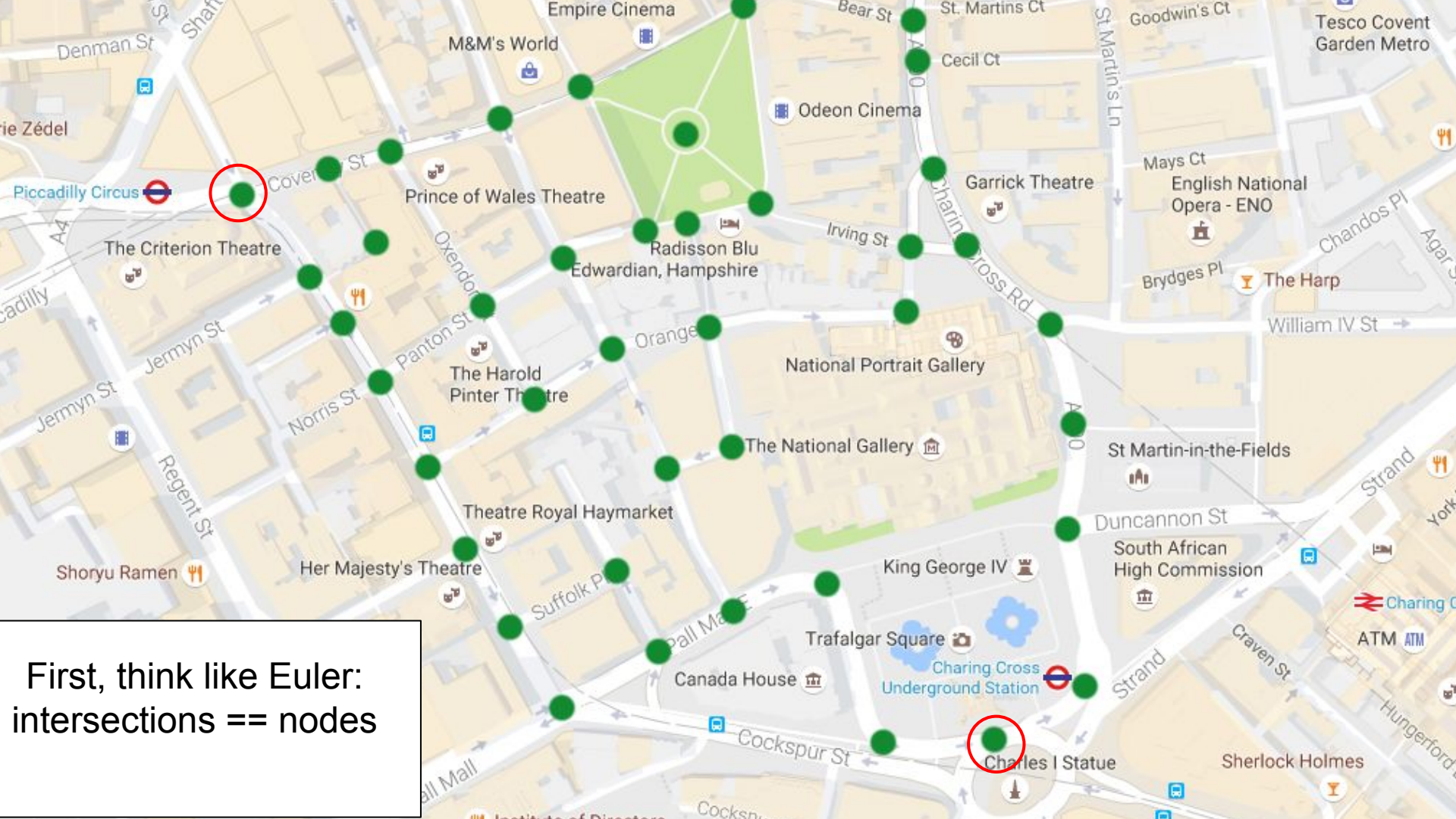
Depth-first search would be more susceptible to its worst-case scenario - it would just get lost without supreme luck!

Breadth-first approaches such as Dijkstra’s algorithm or A* search would fare much better.

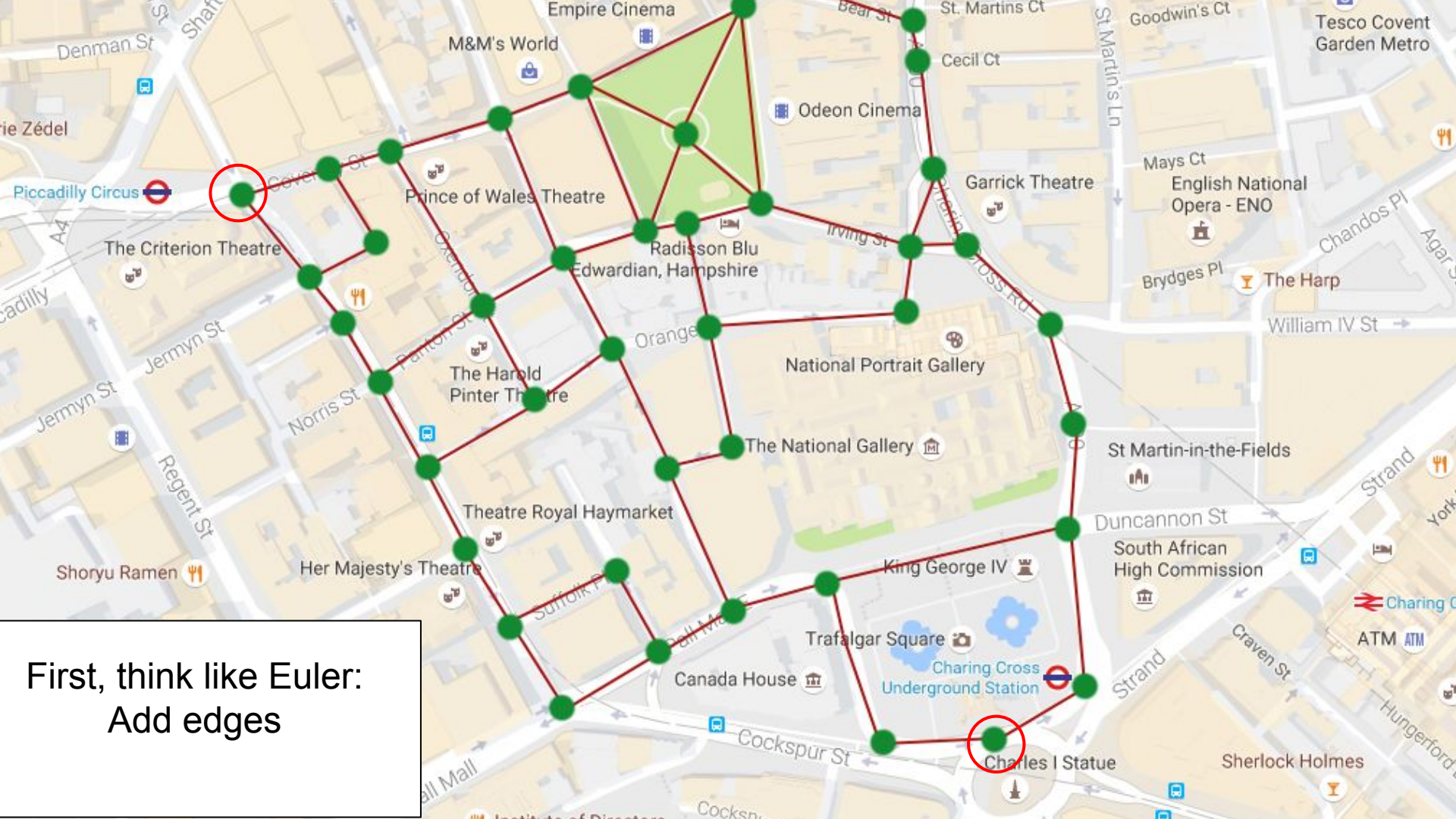
Breadth-first: Dijkstra’s algorithm



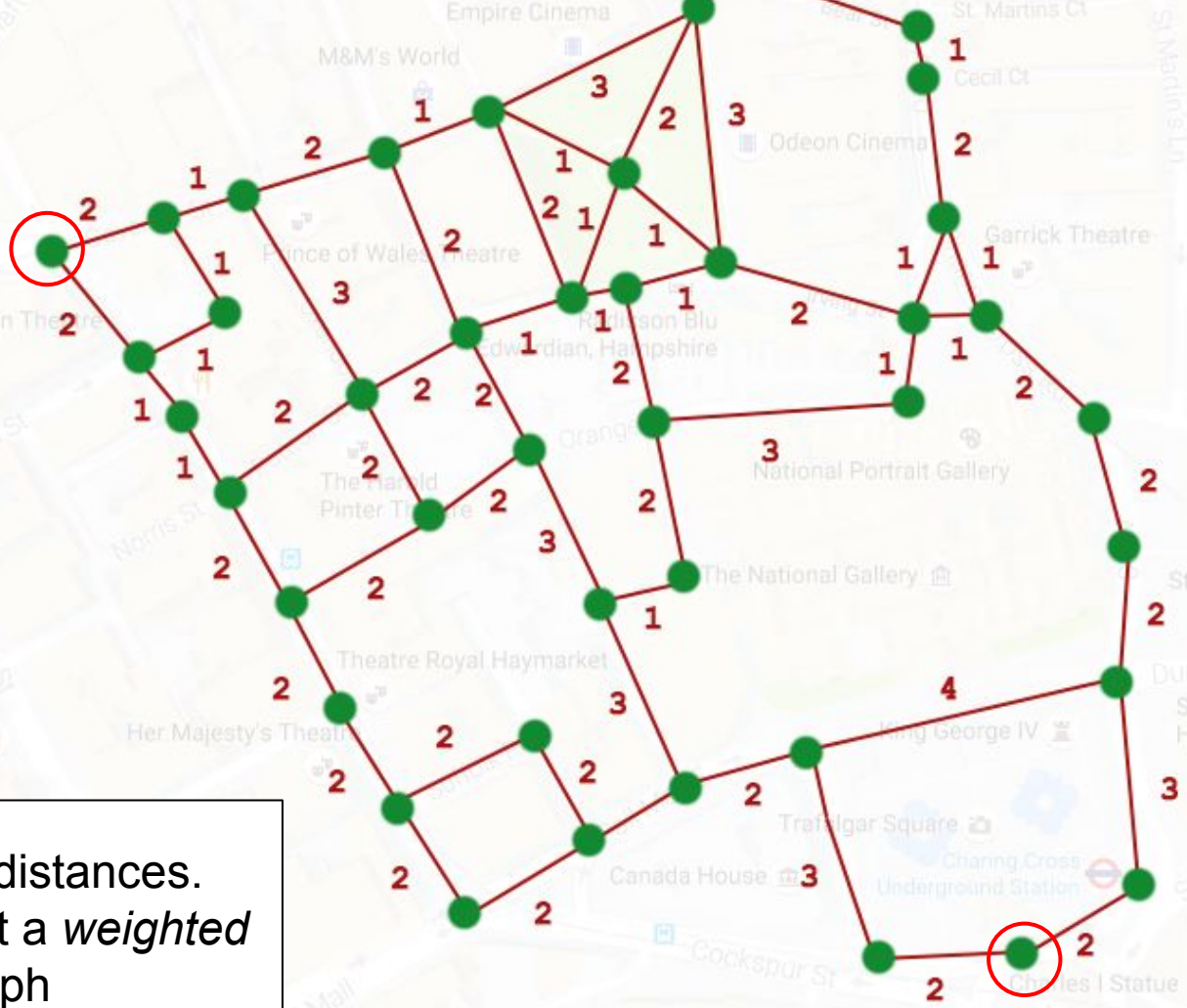
Problem: get from Piccadilly Circus to Nelson's Column



First, think like Euler: intersections == nodes



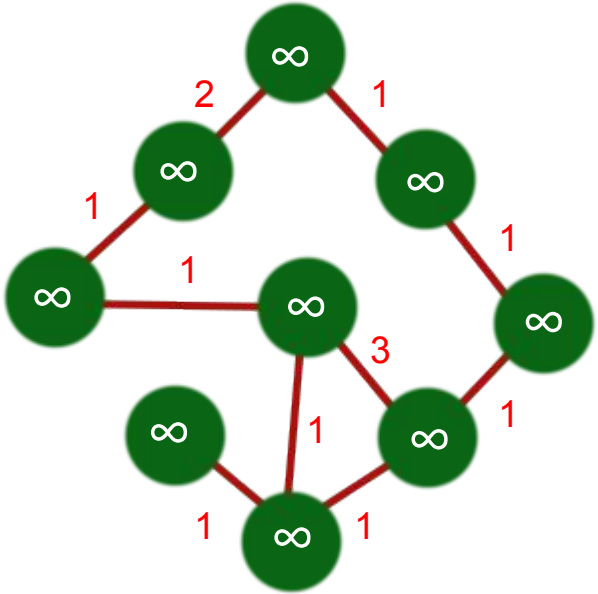
First, think like Euler:
Add edges



Next, add distances.
This makes it a *weighted* graph

Dijkstra’s algorithm

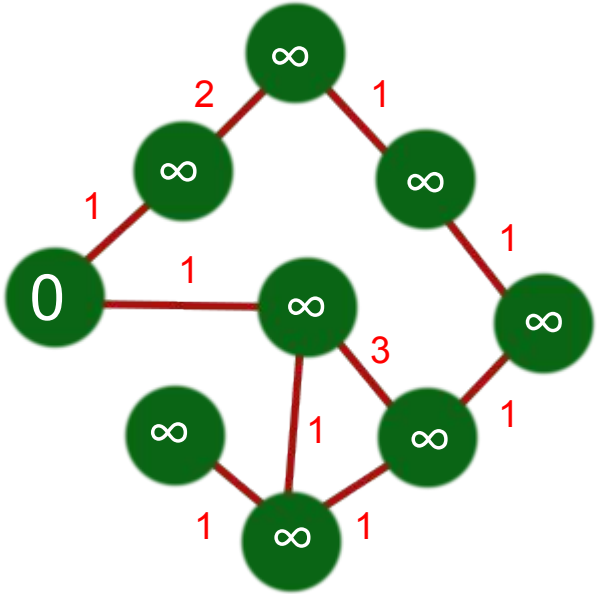
All nodes start with a score of “infinity”



Dijkstra’s algorithm

All nodes start with a score of “infinity”

The start node has score 0



Dijkstra’s algorithm

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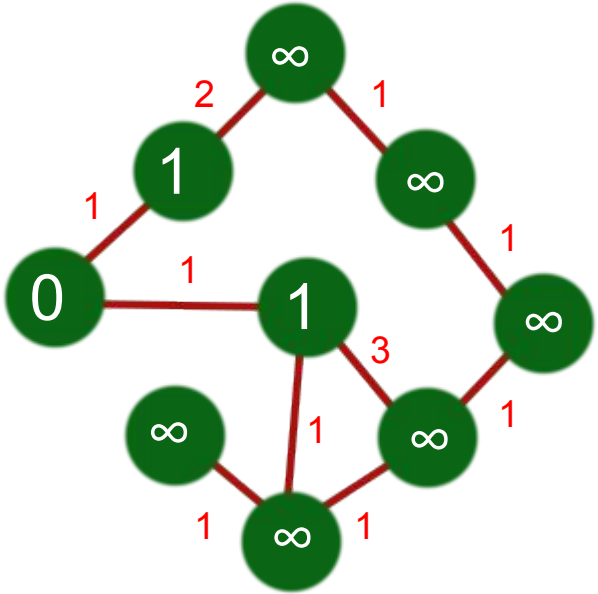
Visit all connected nodes and set their score to node score plus weight if this number is less than current score



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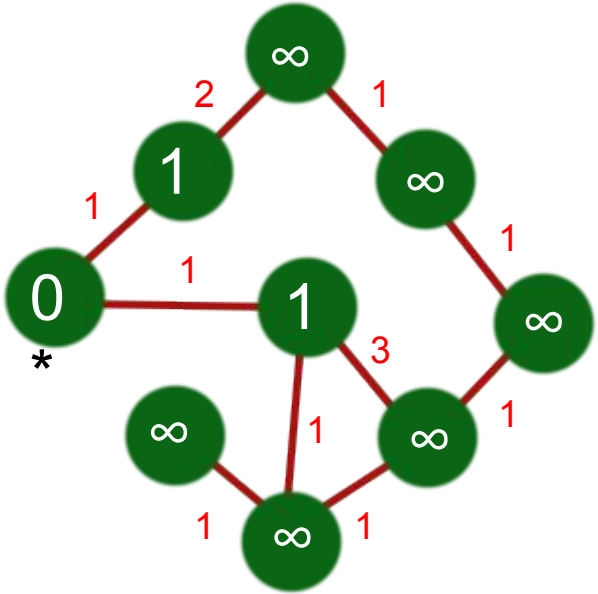
Mark as visited



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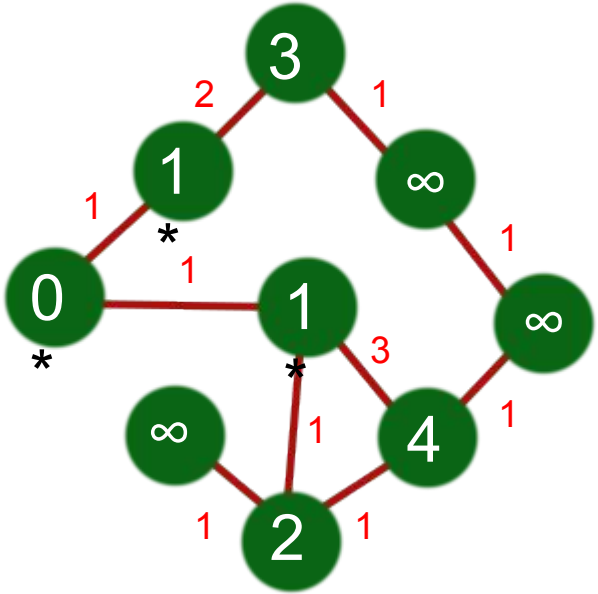
Once all nodes at current depth *with a score* are marked as visited, proceed to next depth.



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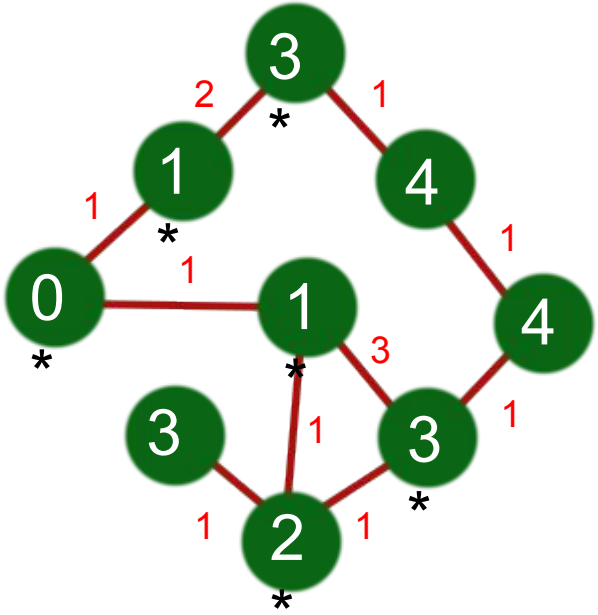
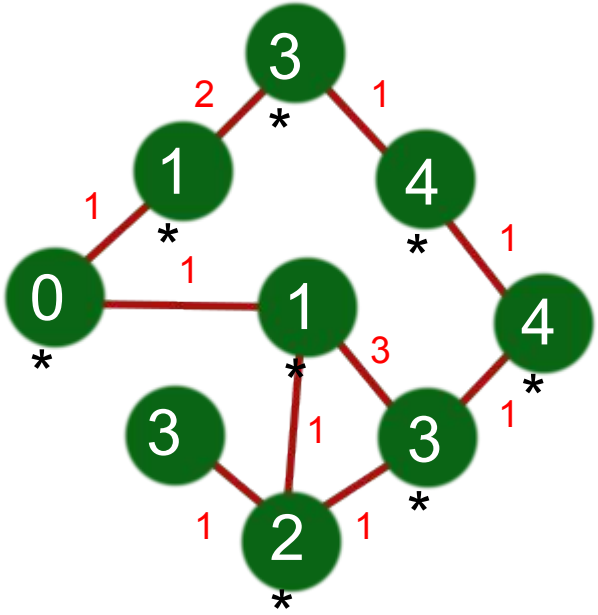


Dijkstra’s algorithm

If the current node is the end node, complete the process and then finish.



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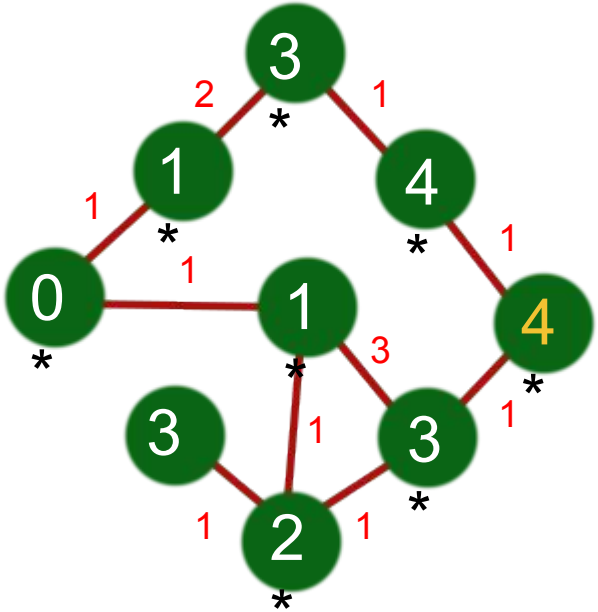
If the current node is the end node, complete the process and then finish.

The shortest path can then be found by visiting the connected nodes with lowest total score and distance.



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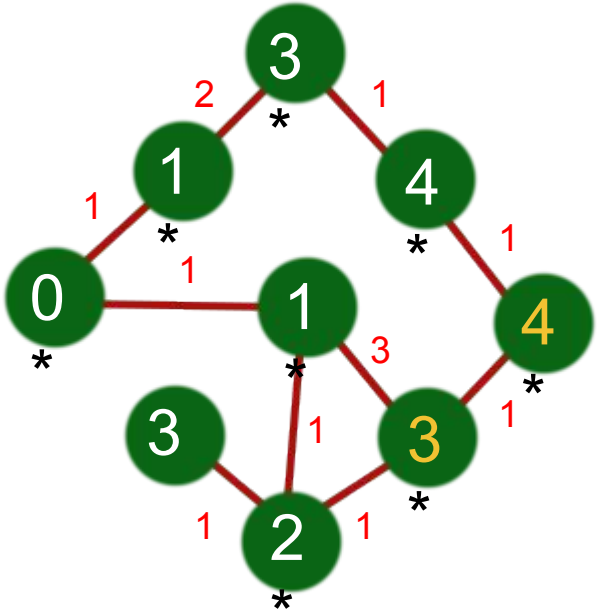

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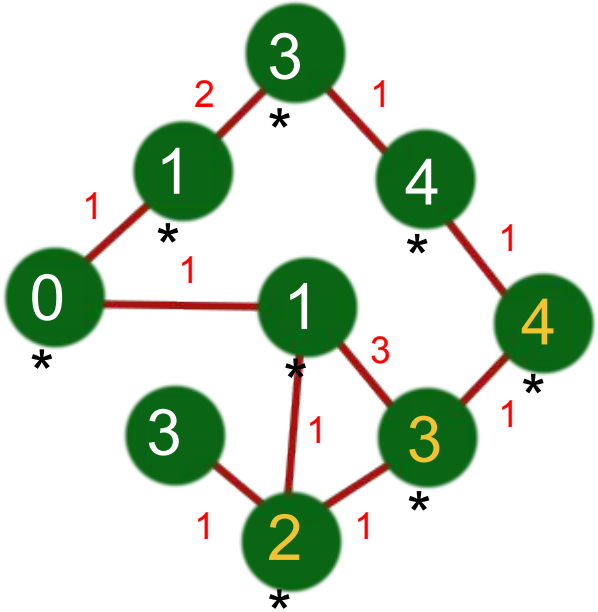

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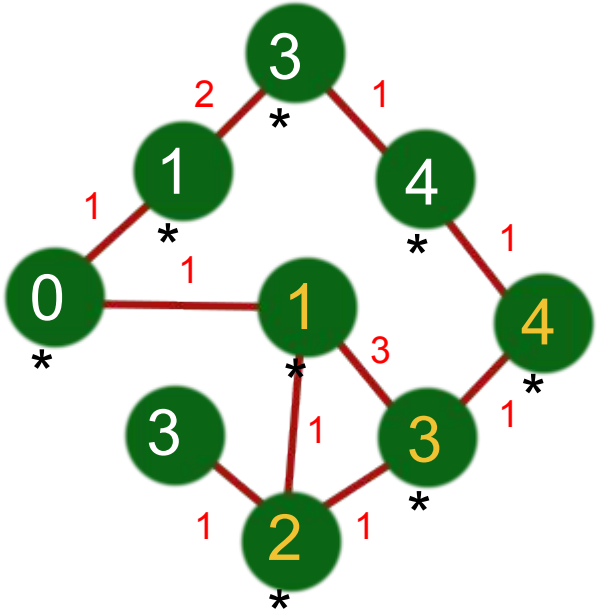

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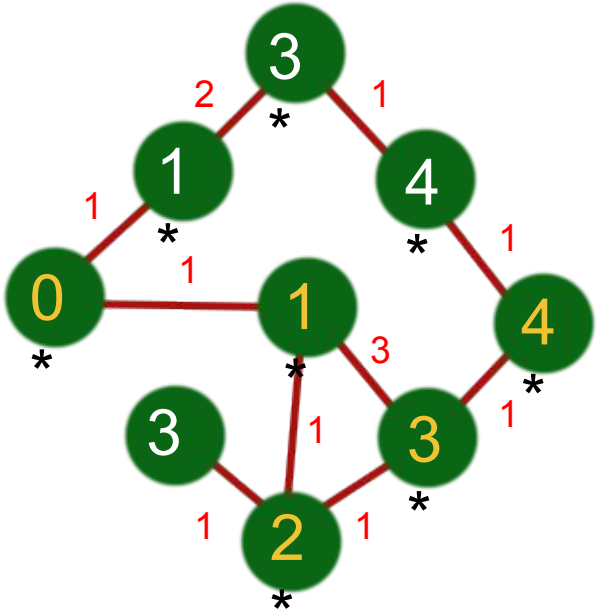

Dijkstra’s algorithm

If the current node is the end node, complete the process and then finish.

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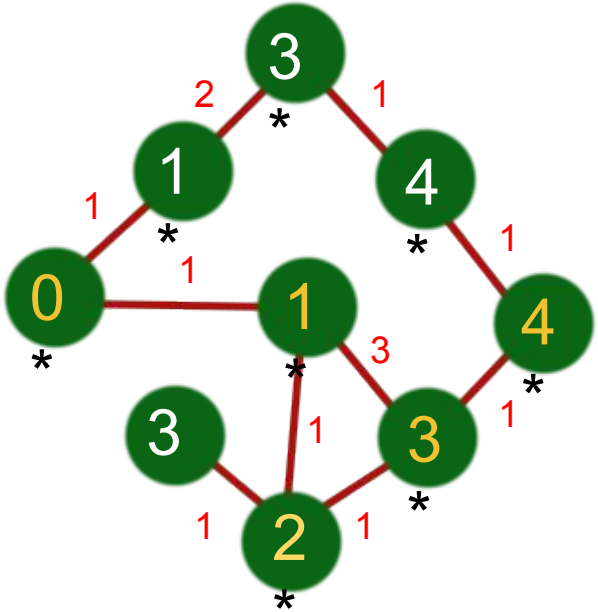
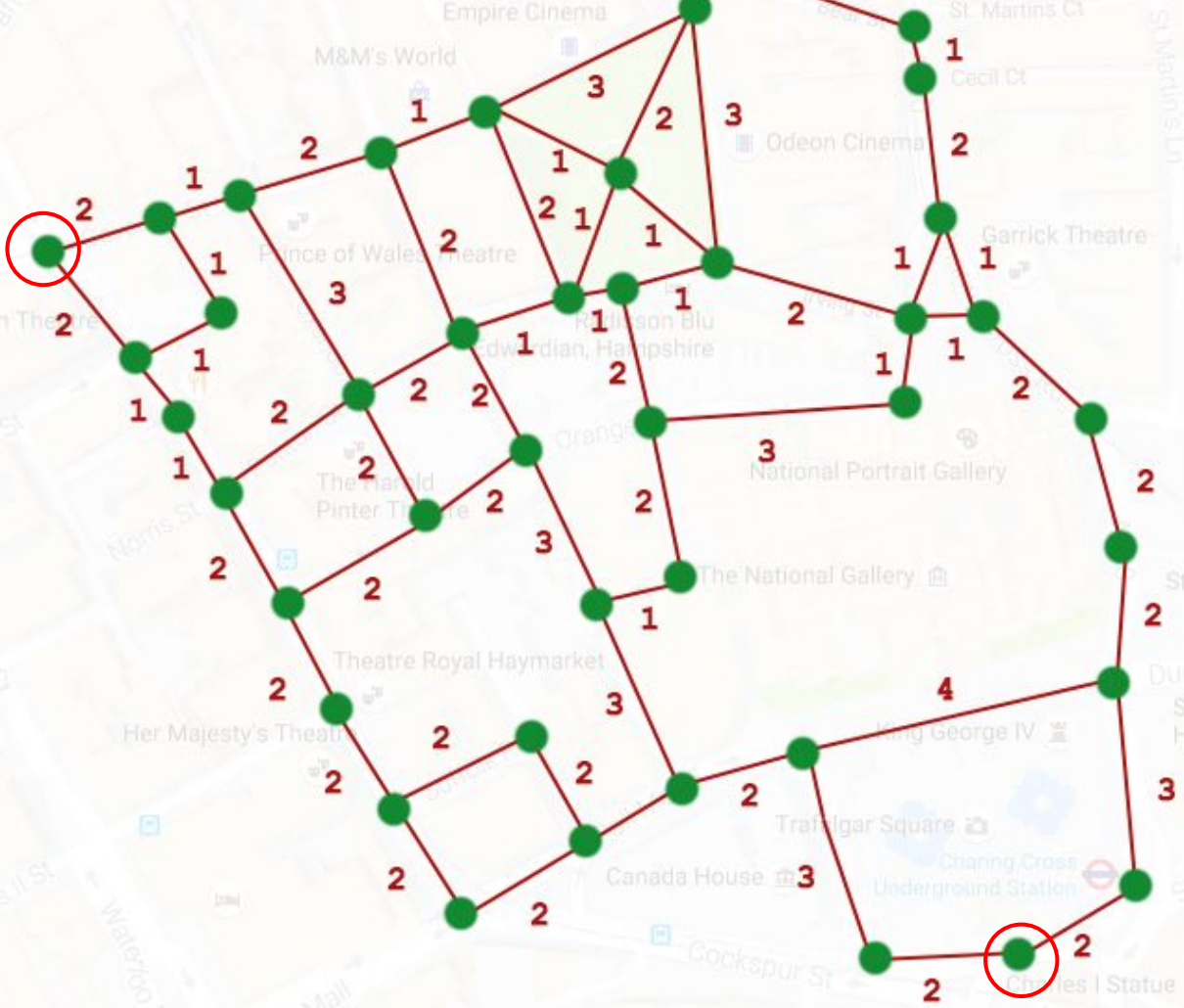
If no nodes with a score are left, and the end has not been found, then there is no path.

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Piccadilly Circus
National Portrait Gallery
The National Gallery
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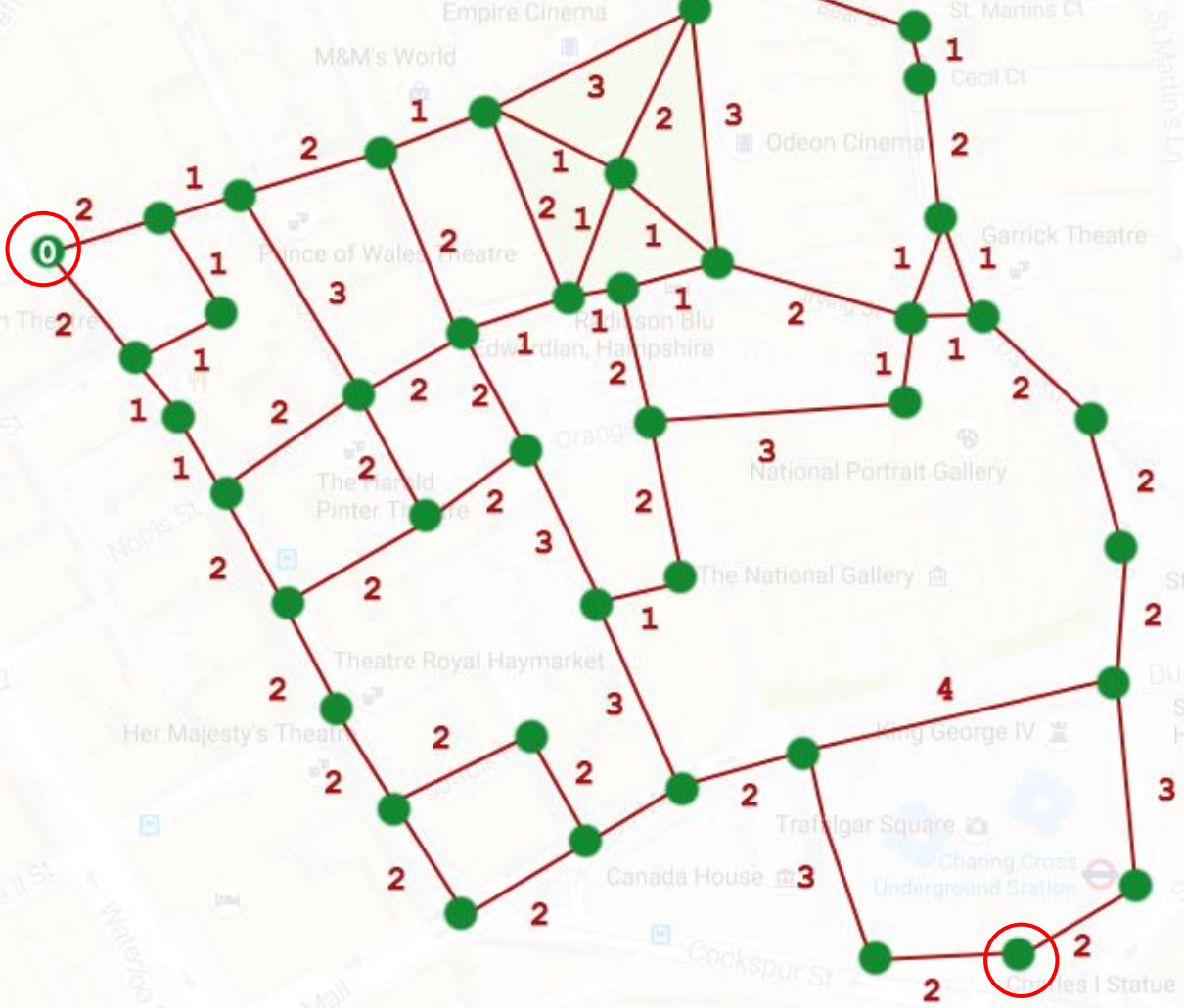
William IV St
Shoryu Ramen
Sherlock Holmes
ATM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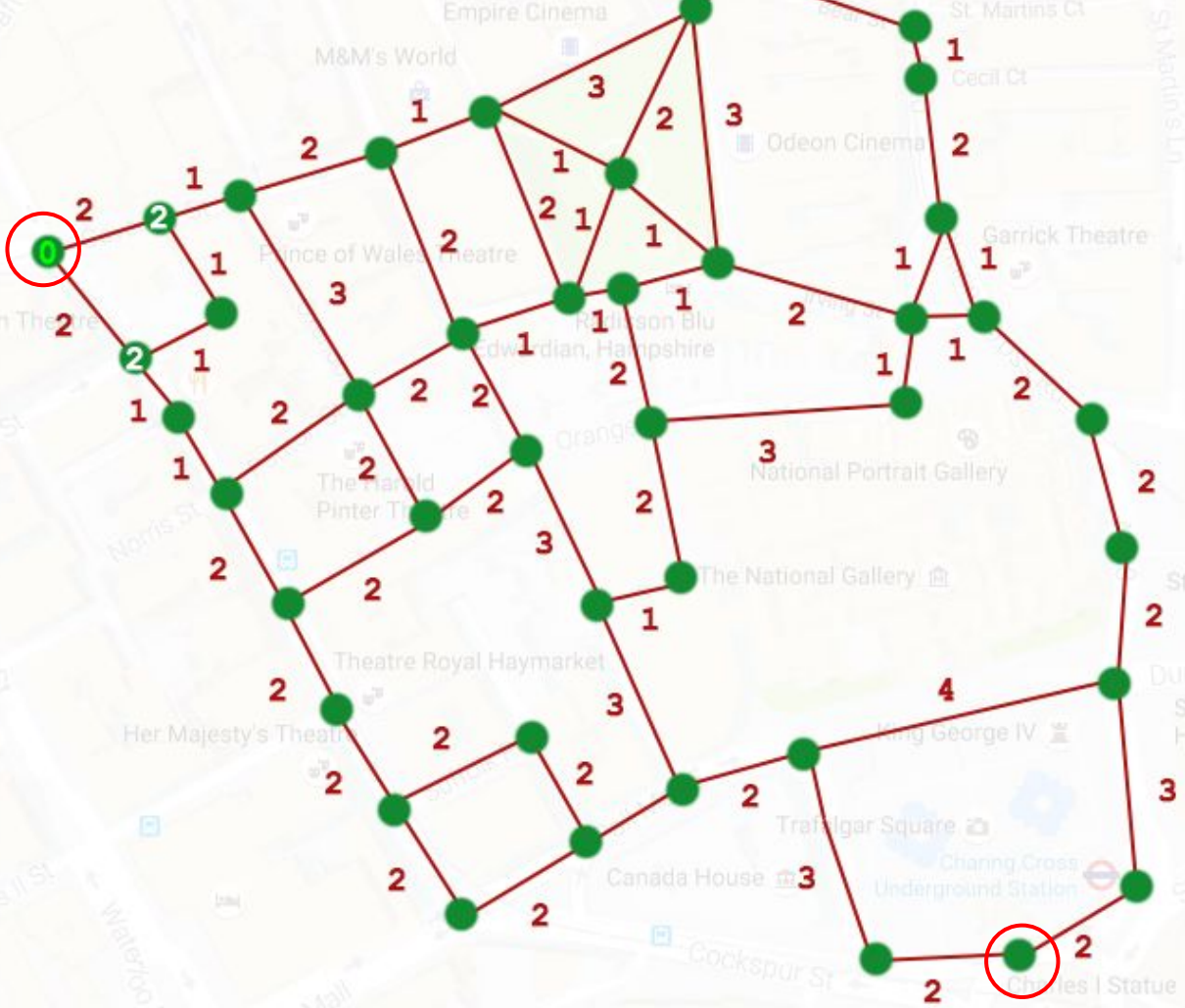
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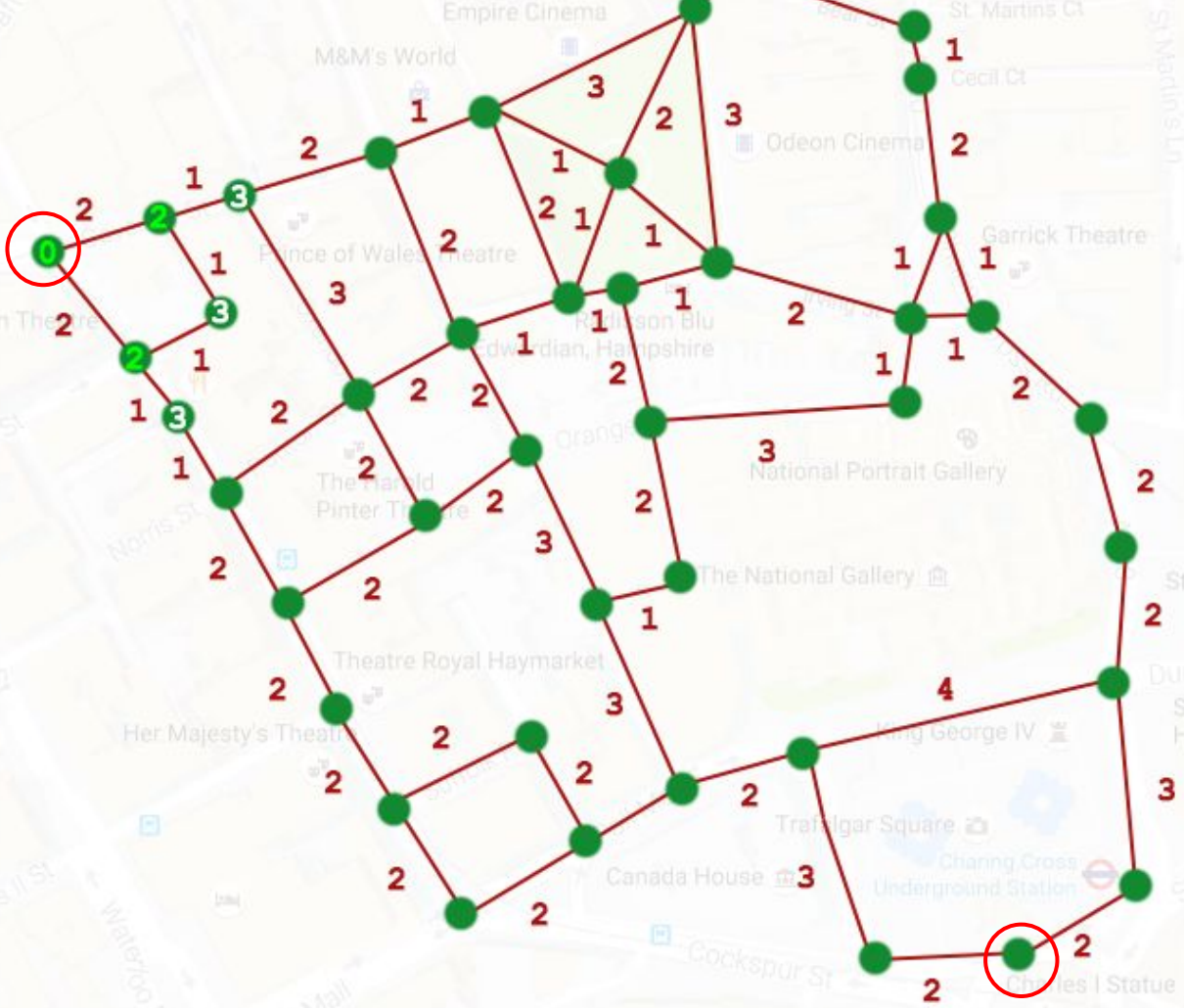
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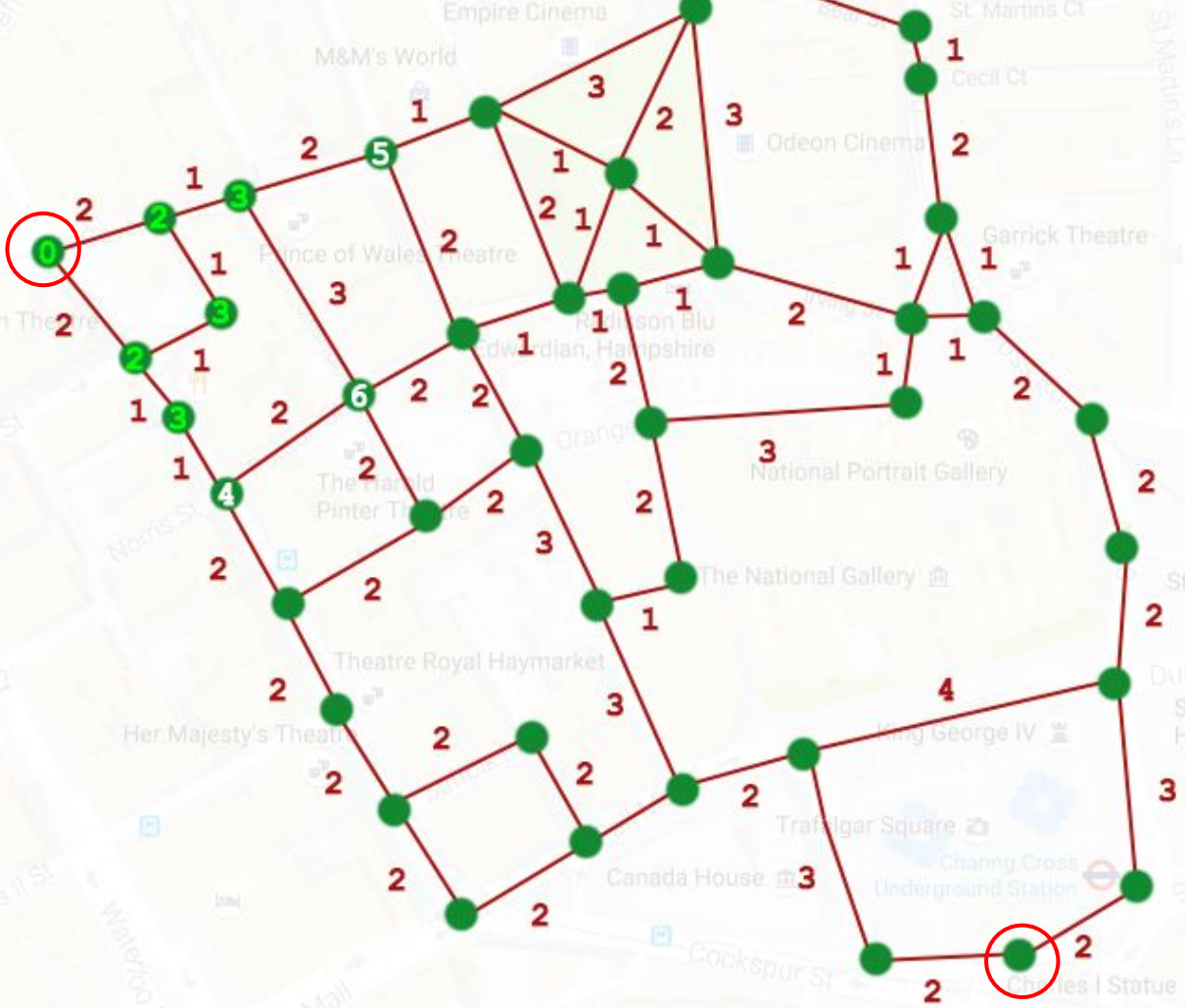
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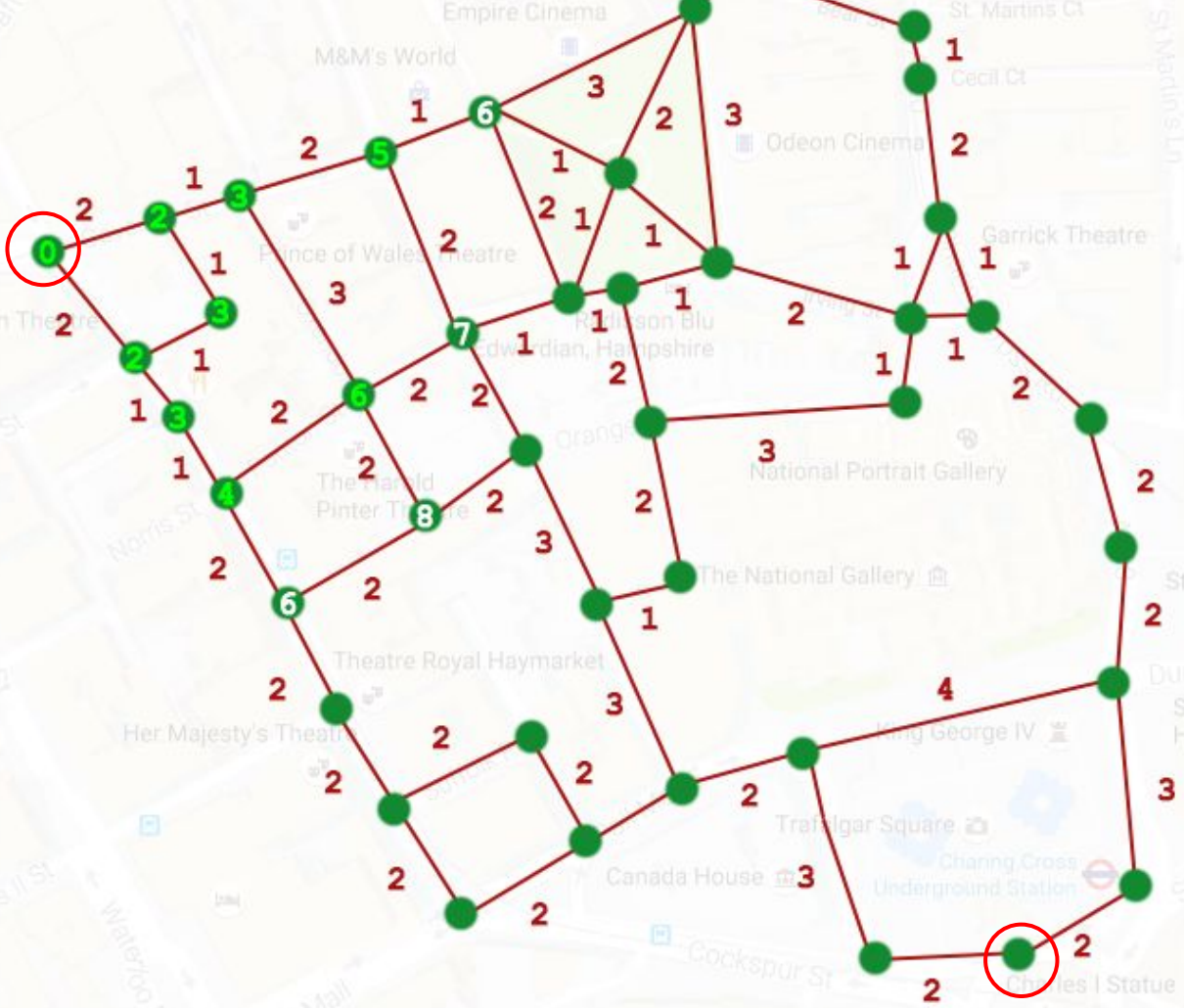
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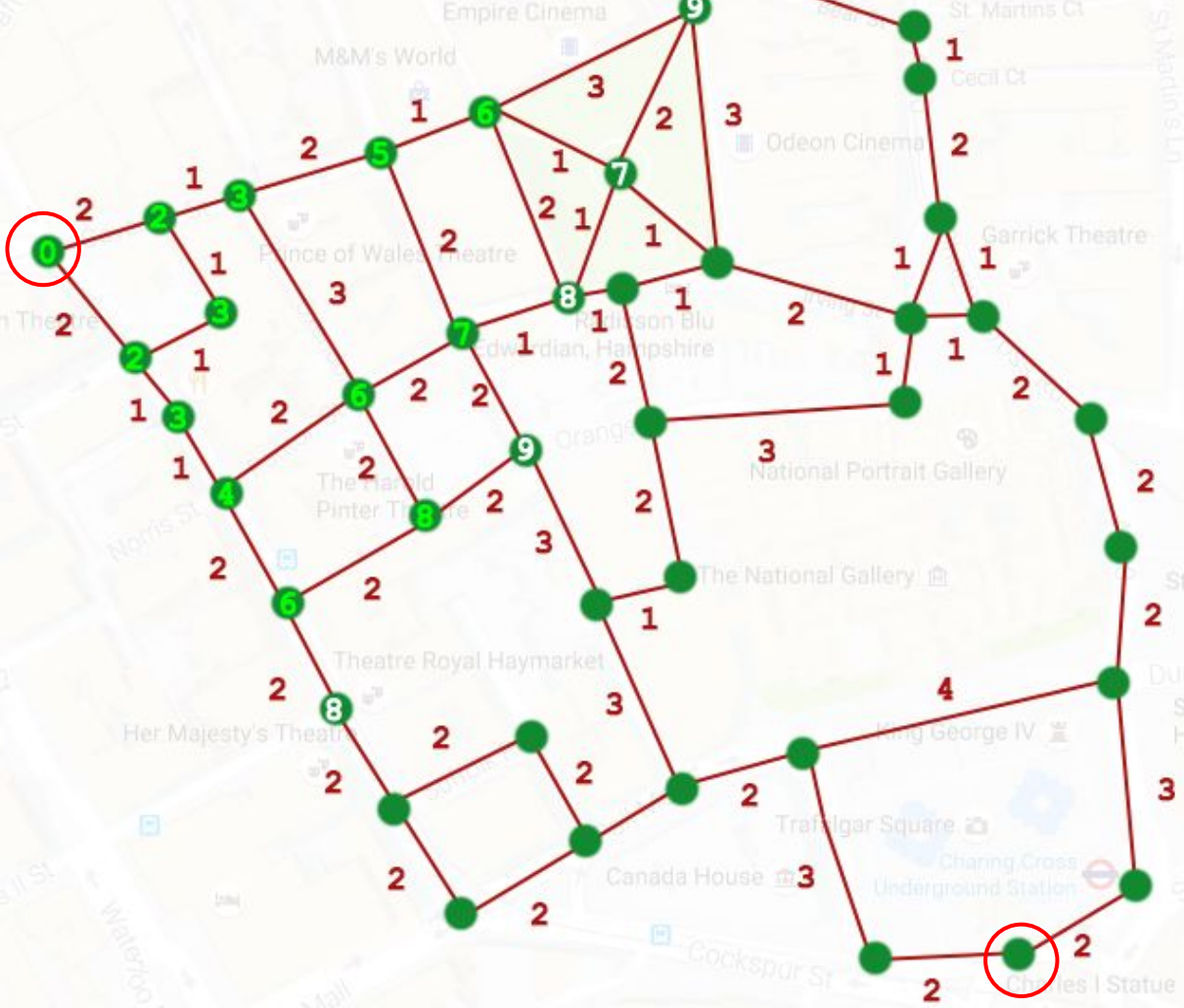
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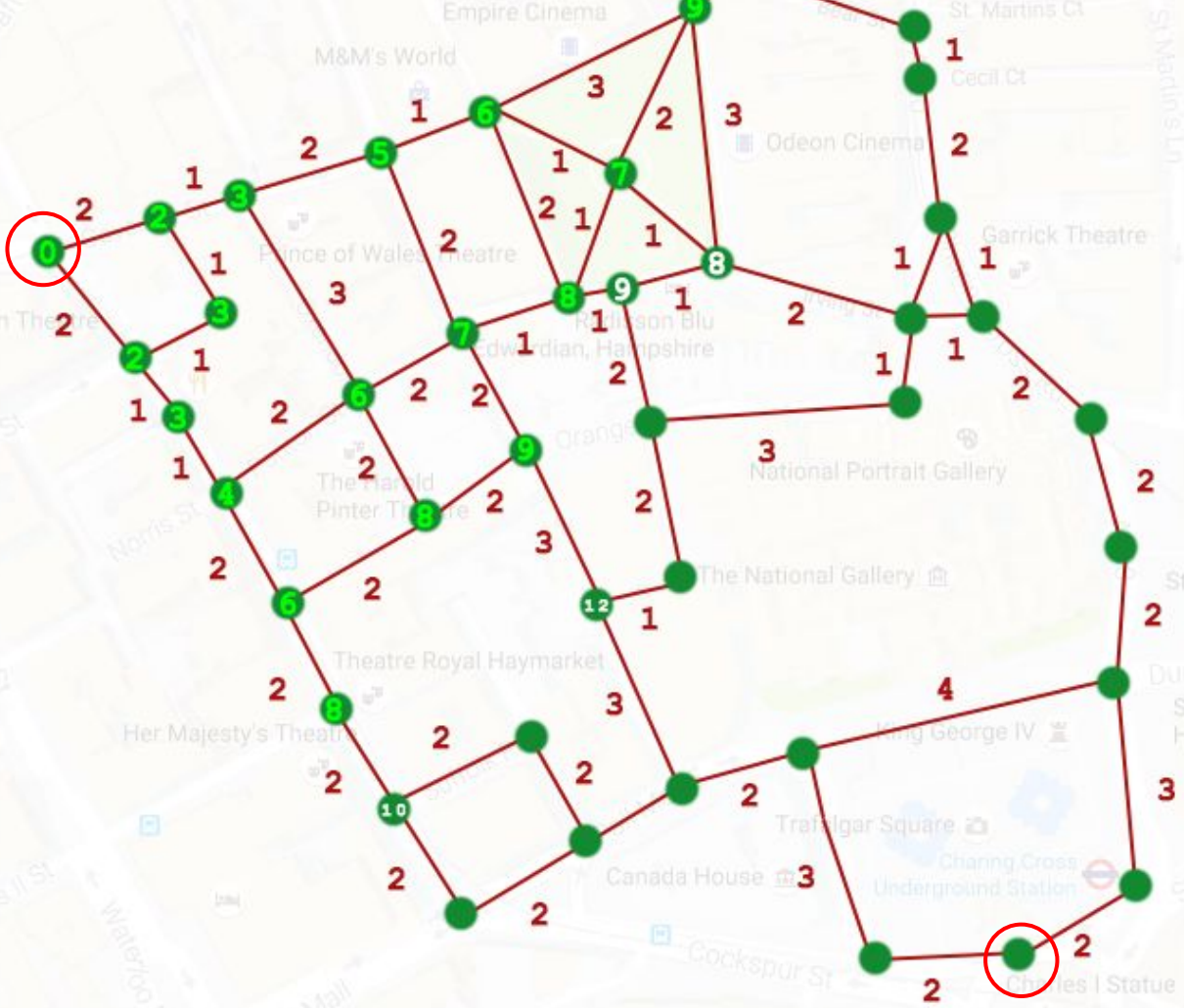
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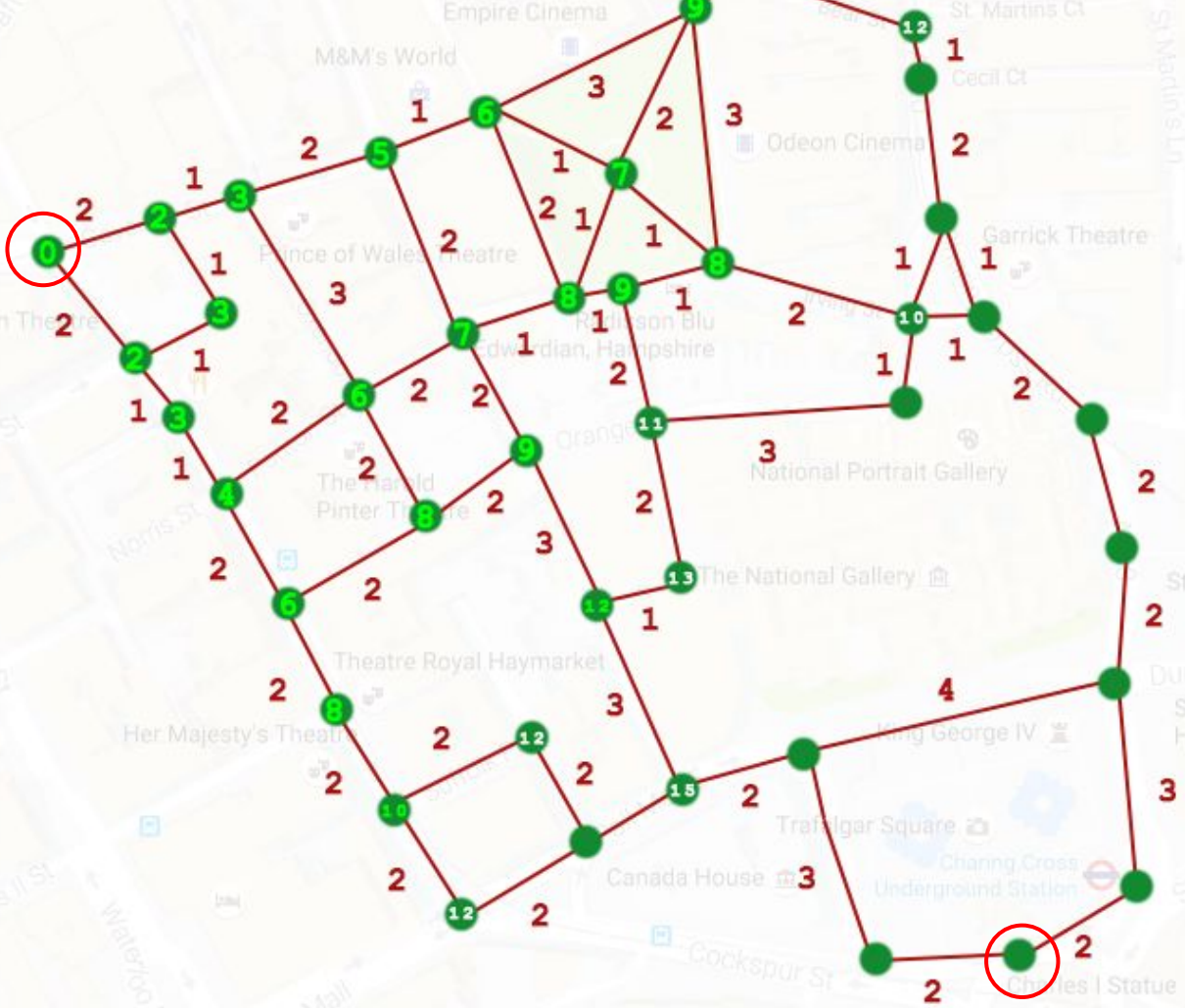
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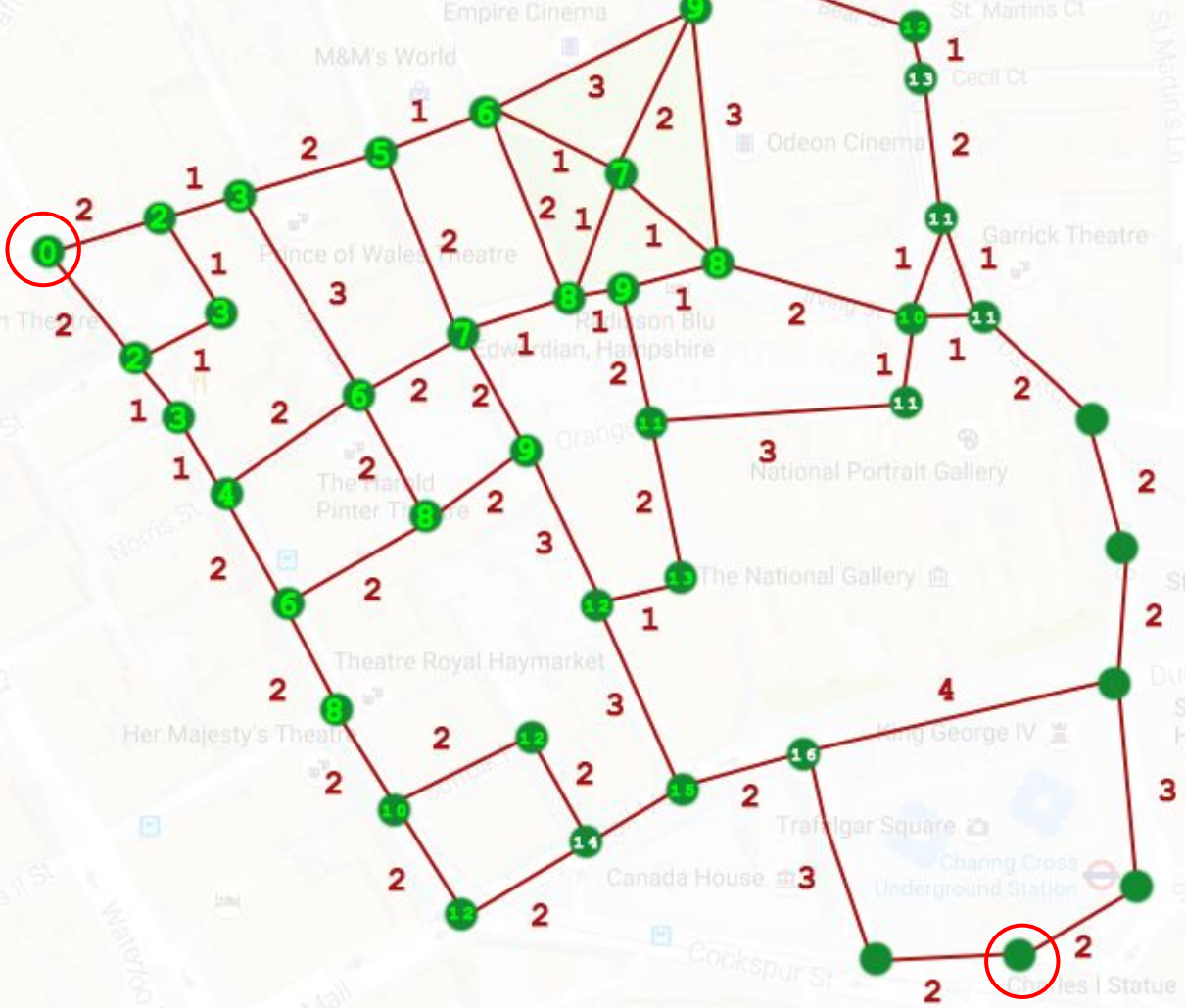
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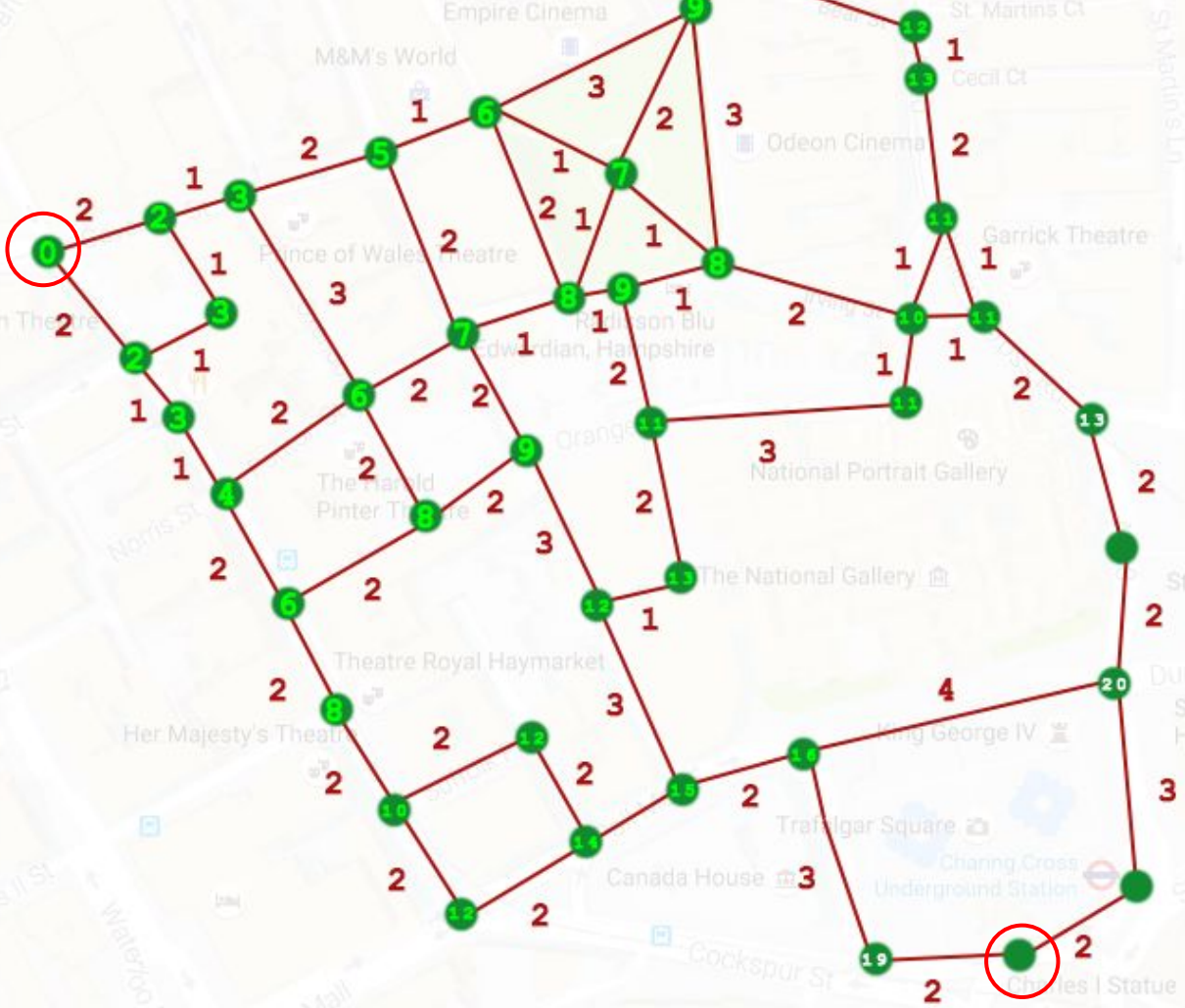
Crawen St
Charing Cross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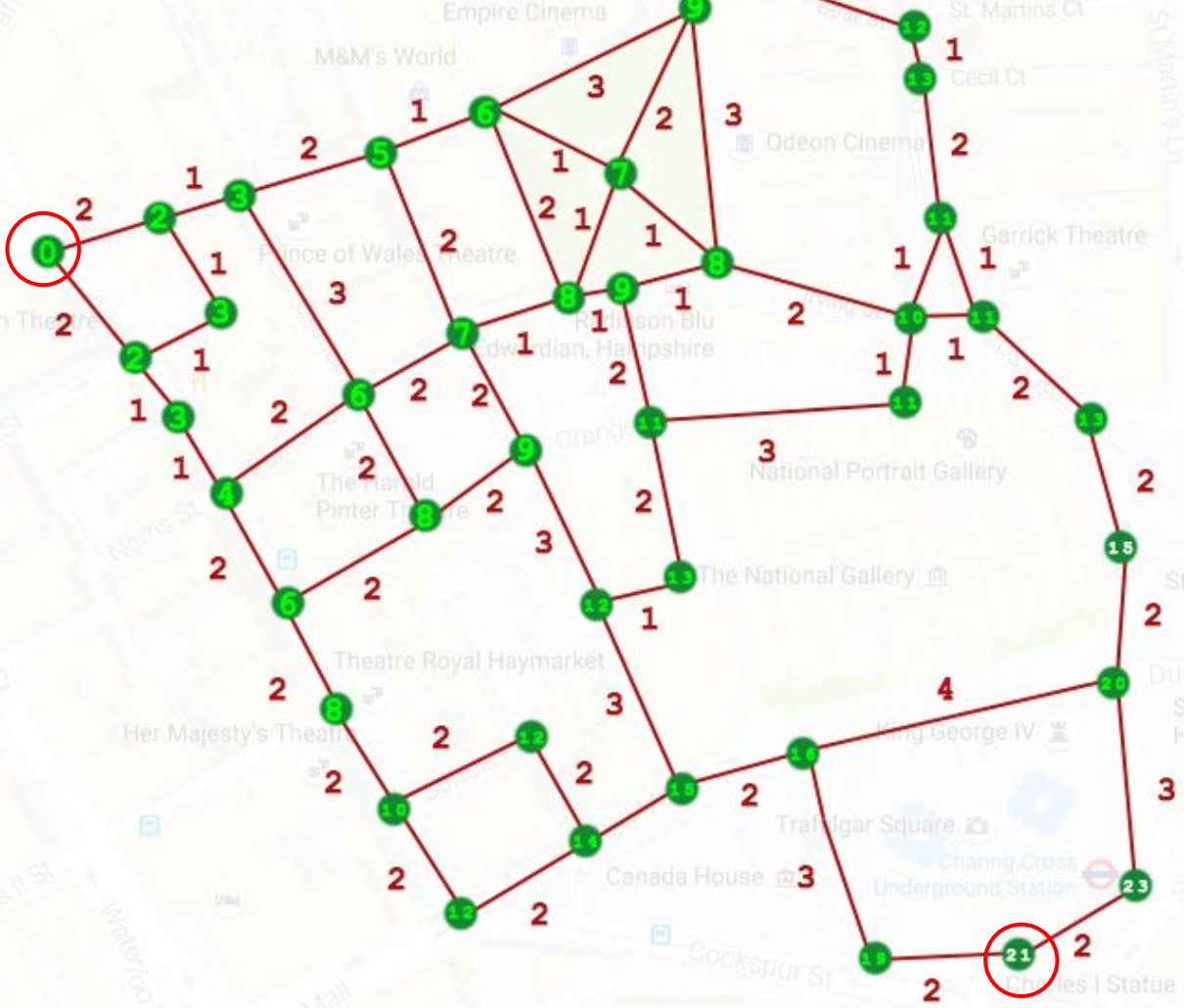
St. Martins Ct
Cecil Ct
Odeon Cinema
Garrick Theatre
Mays Ct
English National Opera - ENO
Tesco Covent Garden Metro
Goodwin's Ct
Denman St
Piccadilly Circus
The Criterion Theatre
Prince of Wales Theatre
Brydges Pl
The Harp
William IV St
National Portrait Gallery
The Harold Pinter Theatre
The National Gallery
St Martin-in-the-Fields
Duncannon St
South African High Commission
Theatre Royal Haymarket
Her Majesty's Theatre
King George IV
Trafalgar Square
Charing Cross Underground Station
Canada House
Charles I Statue
Sherlock Holmes
Shoryu Ramen
ATM
Cockspur St
Pall Mall
Charles II St
Waterloo Pl
Jermyn St
Regent St
Strand
Crawen St
Charing C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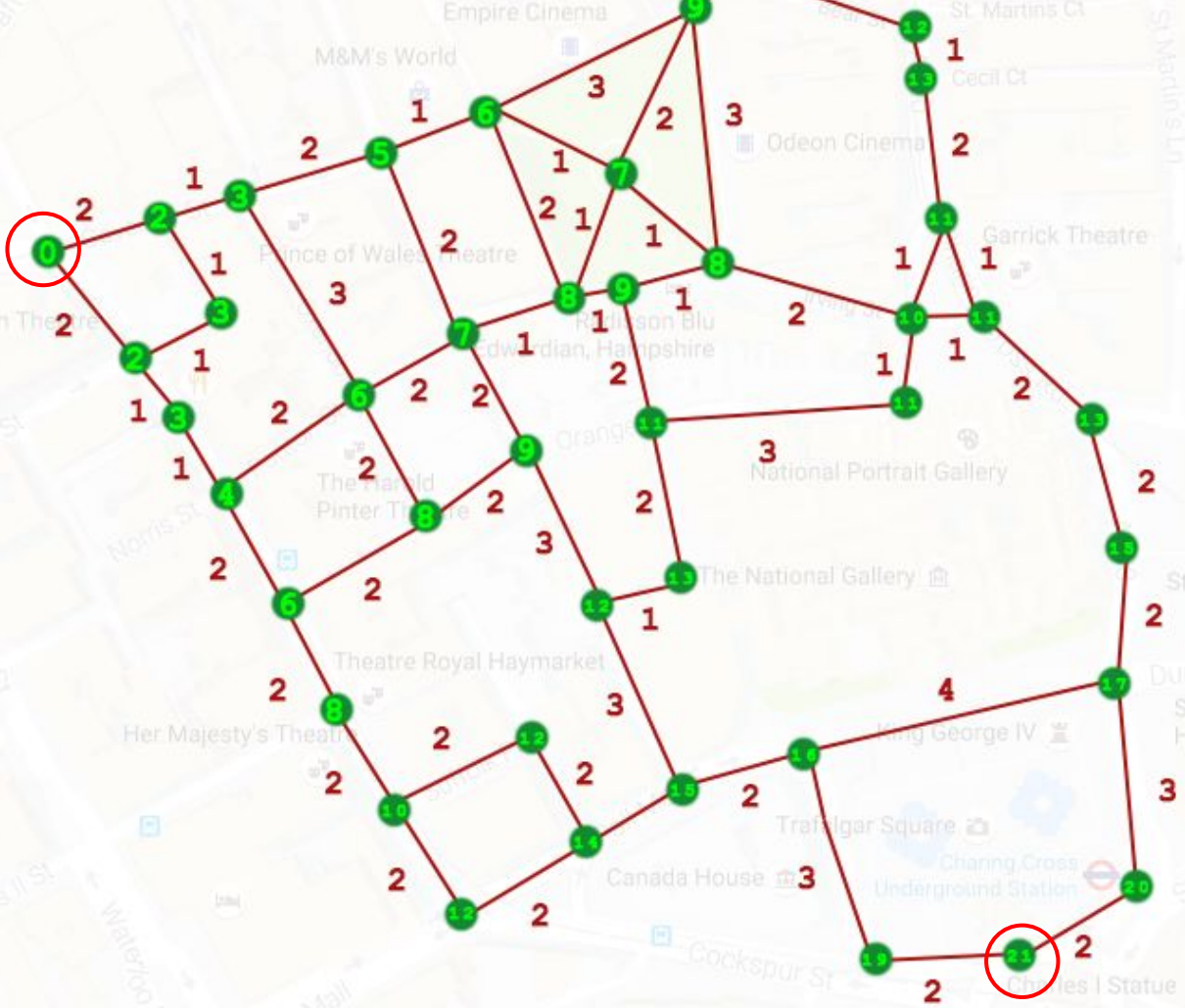
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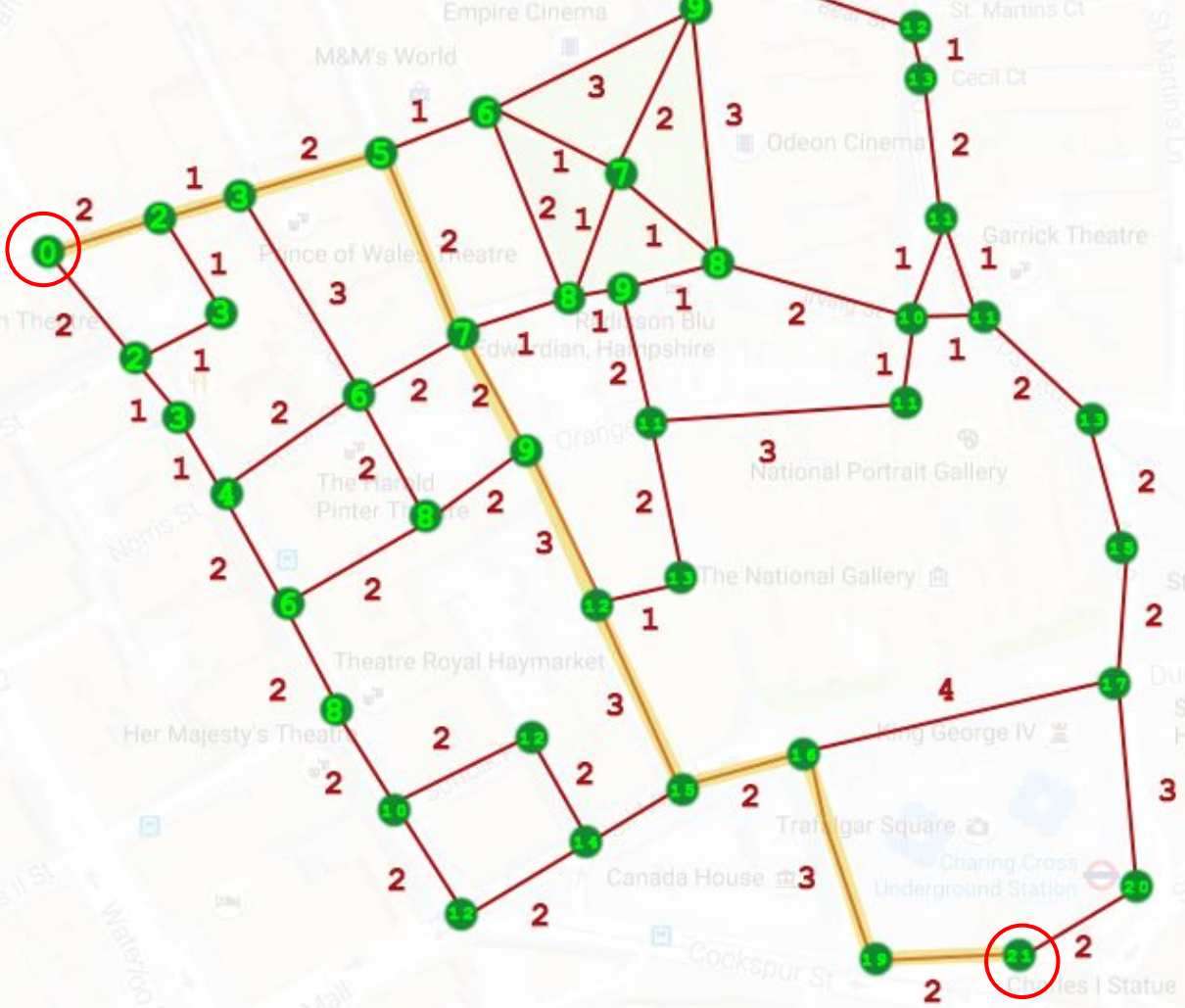
Regent St
Strand
Crawen St
Hungerford
Charing C
Charndos Pl
Adelphi
Denman St
Shaftesbury
St Martin's Ln
Norris St
Jermyn St
Orangery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Useful for modelling physical systems and biased networks.

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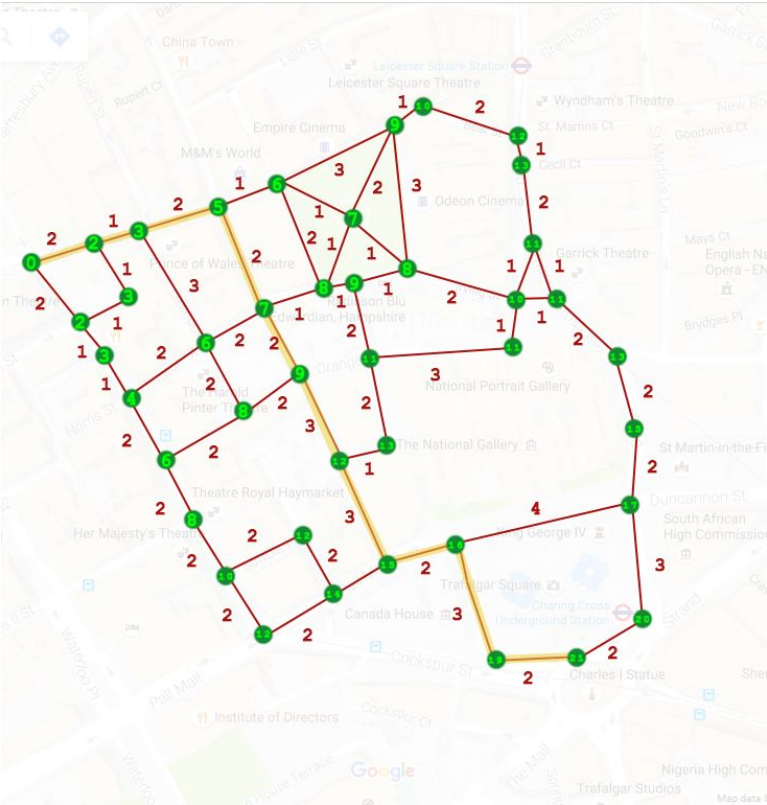

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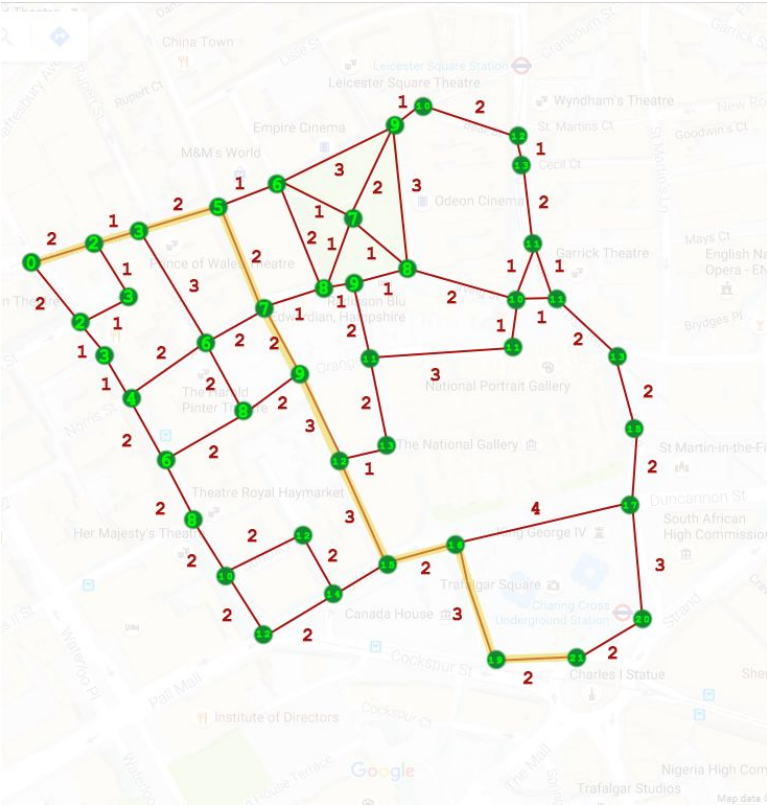
Or perhaps use crime figures to avoid dangerous areas.



Weighted graphs

Useful for modelling physical systems and biased networks.

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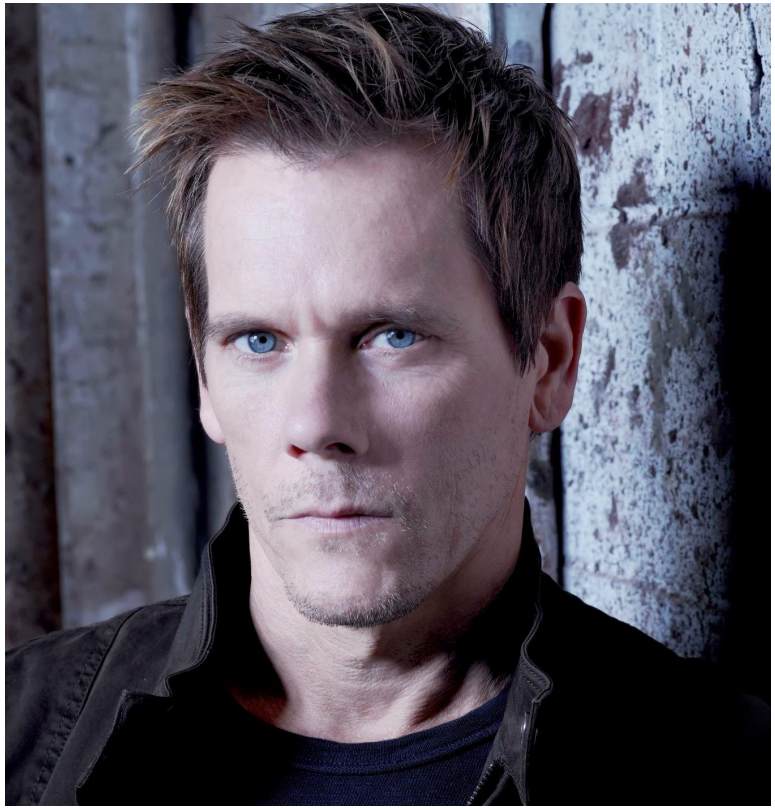
Or perhaps use crime figures to avoid dangerous areas.

We could even use IMDB ratings to weight our Bacon Number search - going via the worst movies.



Trees

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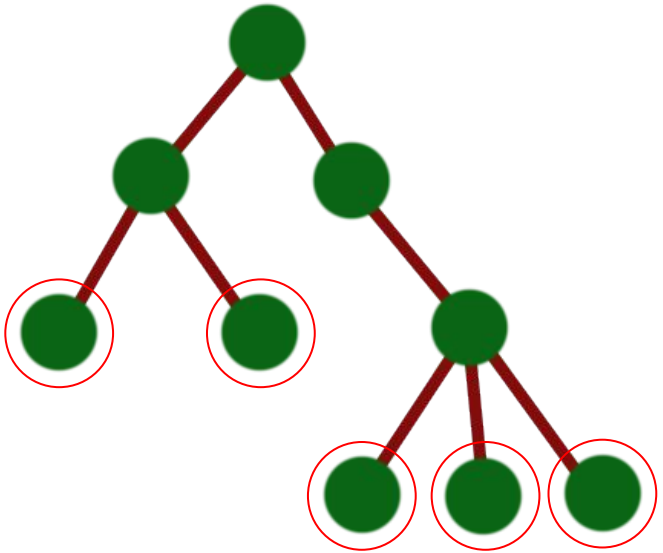
A tree is a special kind of graph; it still has nodes and edges.

Where a node has a degree of one it is called a *leaf*.



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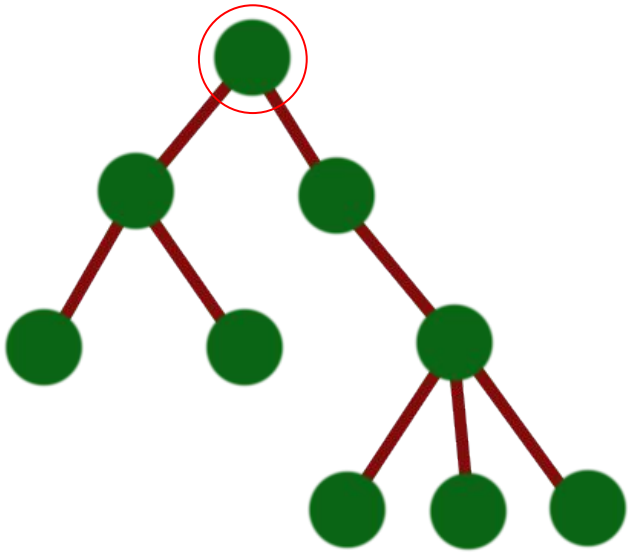

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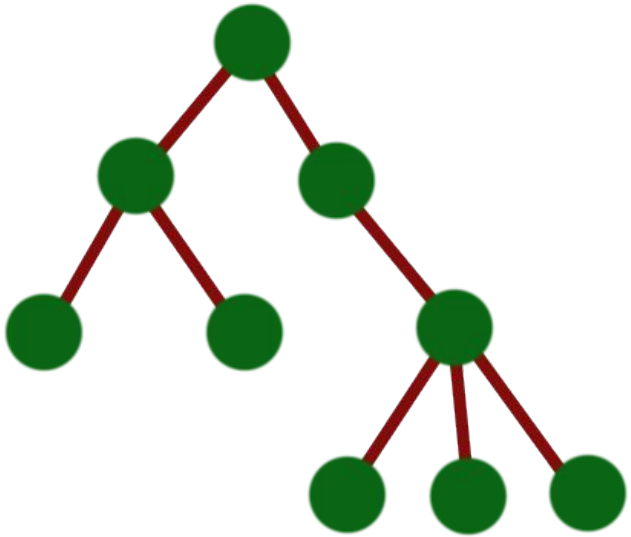

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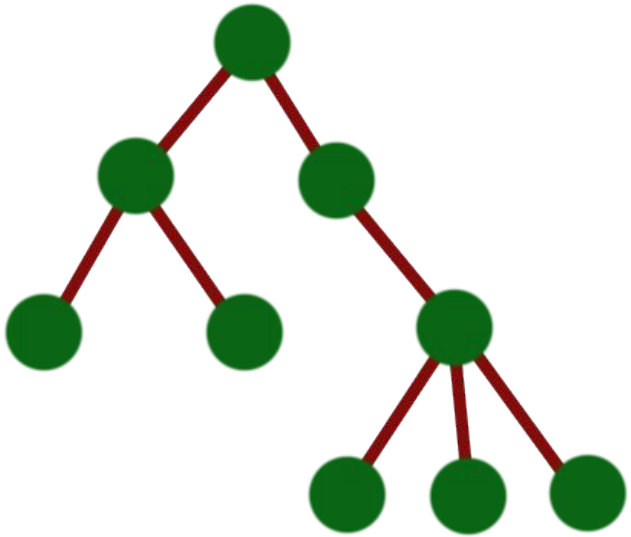
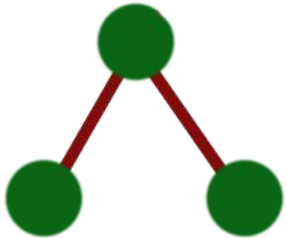
It is an example of an Acyclic Graph - it has no cycles.



Trees

Much like real trees, they are *self-similar*



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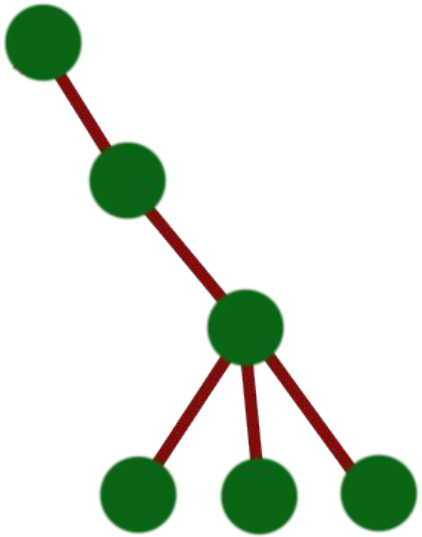
When you look at smaller pieces, they look a lot like the bigger piece.



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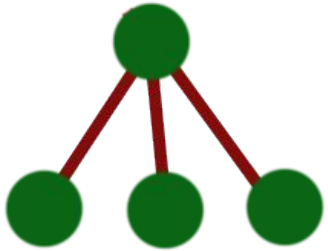
A tree with depth greater than one is a *forest* of other trees.



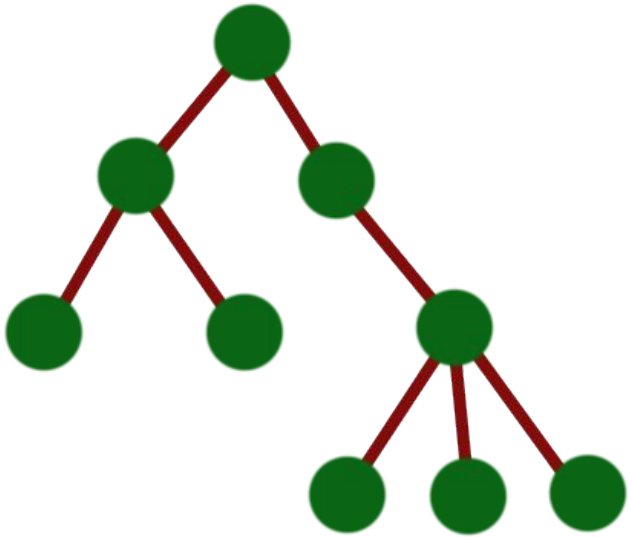
Trees

Much like real trees, they are *self-similar*

When you look at smaller pieces, they look a lot like the bigger piece.

A tree with depth greater than one is a *forest* of other trees.

Therefore any operation you perform on a small tree can be performed on any tree.



Walking trees: Arithmetic


HEWLETT · PACKARD
Continuous Memory
ON
PRGM
RUN
BST
GSB
RTN
GTO
LBL
STO
DSZ
RCL
ISZ
CHS
GRD
EEX
RAD
CLX
DEL
Σ+
ENTER ↑
PREFIX
CLEAR PRGM
REG
f
g
x<y
R↓
R/S
PAUSE
LAST x
→H.MS
→H
INT
FRAC
ln
e^x
sin
sin⁻¹
log
10^x
cos
cos⁻¹
tan
tan⁻¹
→R
→P
y^x
ABS
√x
x²
1/x
π
x=0
x≠0
x>0
x≥0
x<0
x≤0
DEG

Reverse Polish Notation

The first pocket calculators used RPN to input arithmetic.

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Instead of typing “one plus two equals” you’d type “one two plus equals.”

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The reason for this is rooted in how computers process arithmetic.

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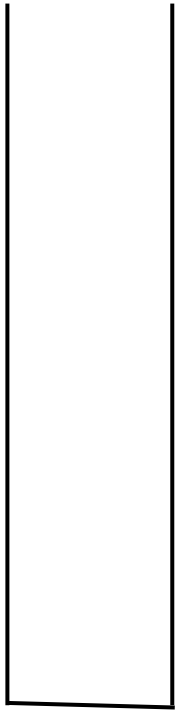
Essentially, stack arithmetic uses a queue and a stack.



5 6 + 3 *

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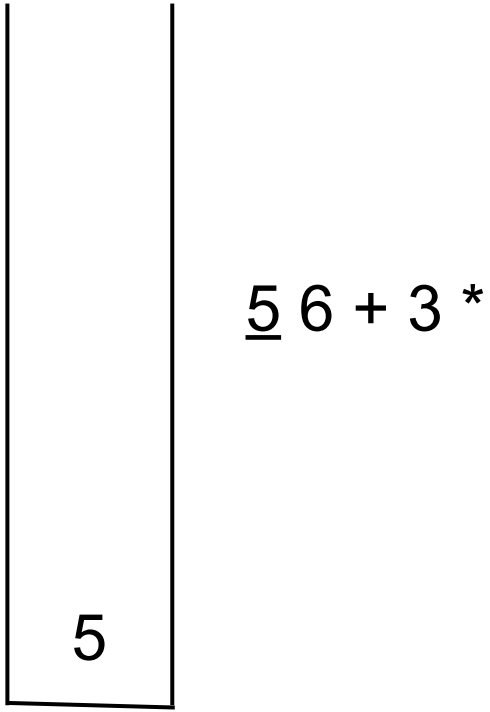
Read each item from the queue in turn.



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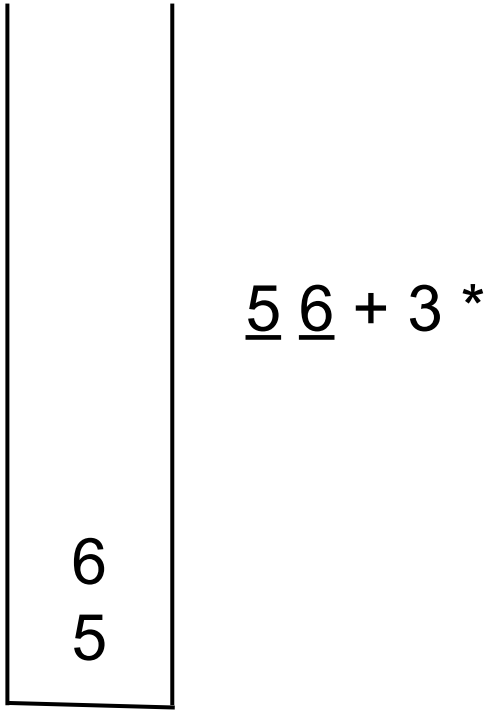

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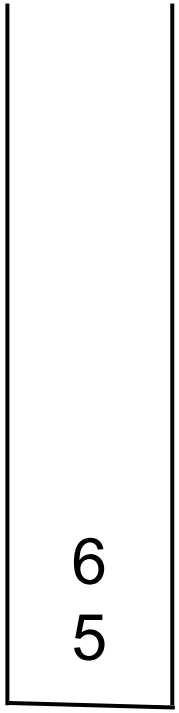

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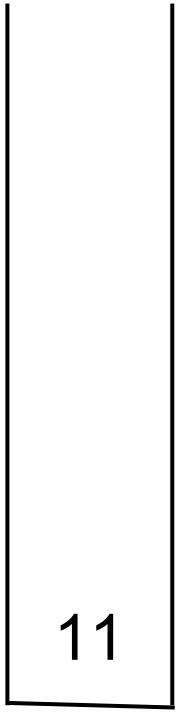

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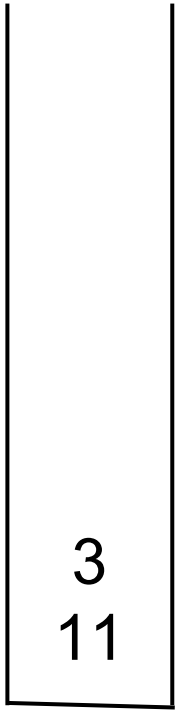

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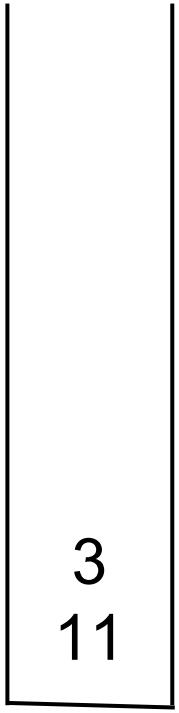

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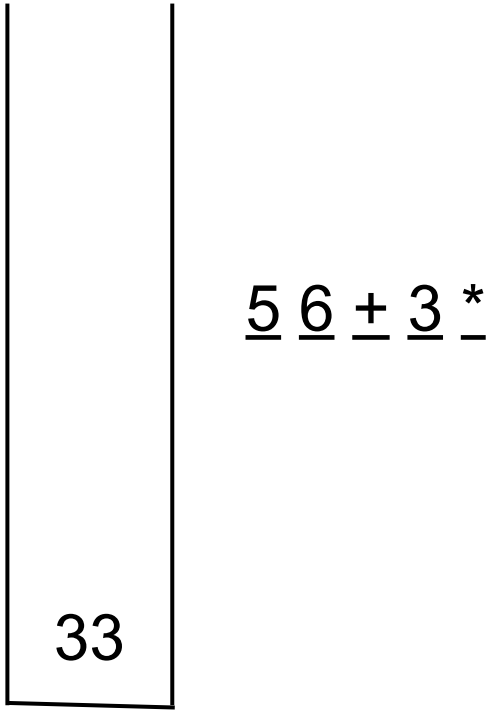

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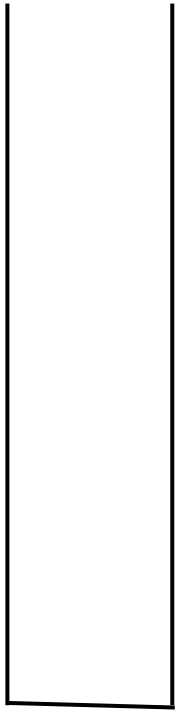
When your queue is finished, you have your answer!



5 6 + 3 * = 33

Reverse Polish Notation

Another property of RPN is there is no need for operator precedence.

3 5 7 + * 23 / 7 +

Reverse Polish Notation

Another property of RPN is there is no need for operator precedence.

But: RPN is pretty illegible!

3 5 7 + * 23 / 7 + = ?!

Reverse Polish Notation

Another property of RPN is there is no need for operator precedence.

But: RPN is pretty illegible!

How can we transpose it to “natural”?

(Not that it’s much better in this example!)

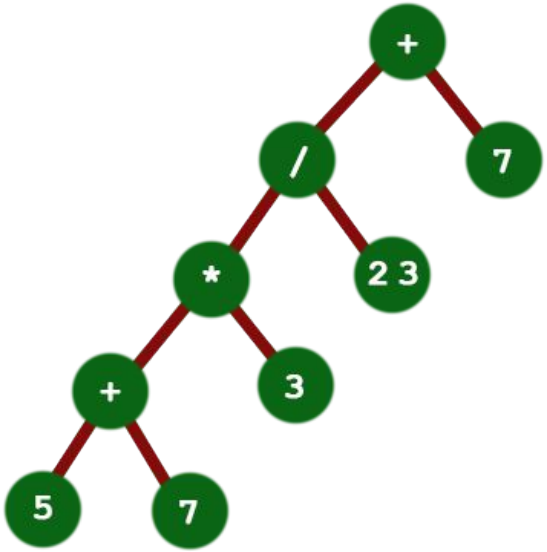
3 5 7 + * 23 / 7 + = ?!

(((5 + 7) * 3) / 23) + 7 = ?

Reverse Polish Notation

Answer: TREES!

(Bet you didn’t see that coming!)



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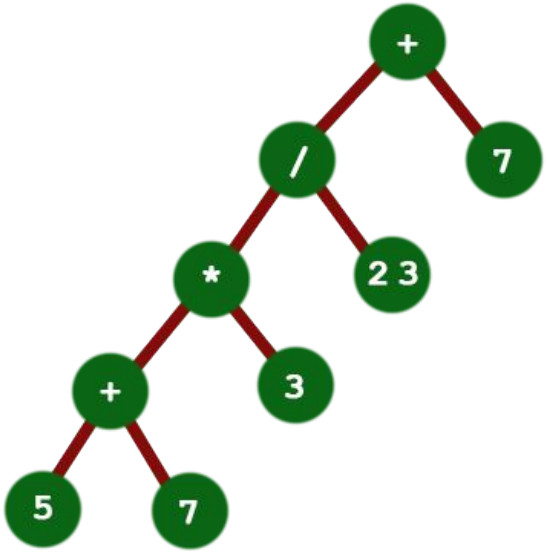

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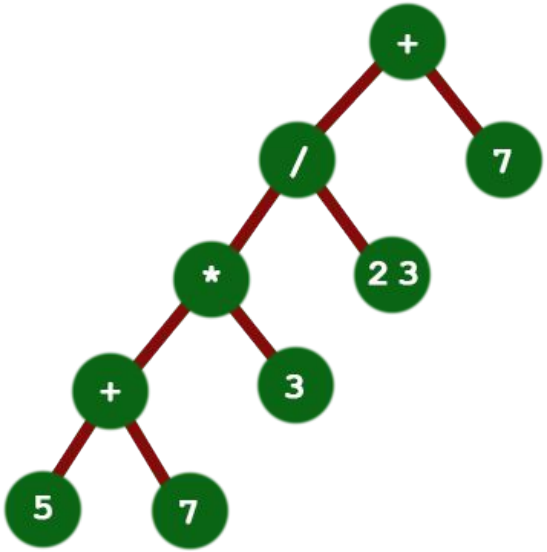
Operators become nodes; operands become leaves.

We can then walk the tree to produce our output. There are three methods.



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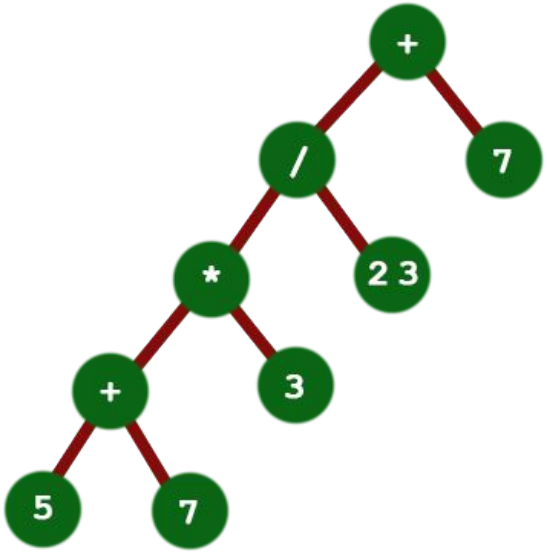

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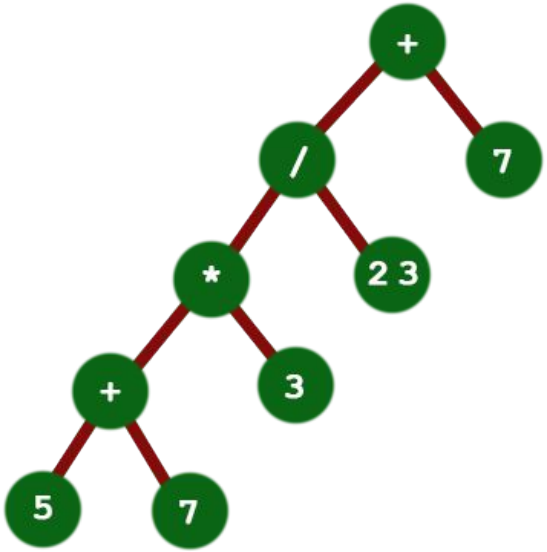

+

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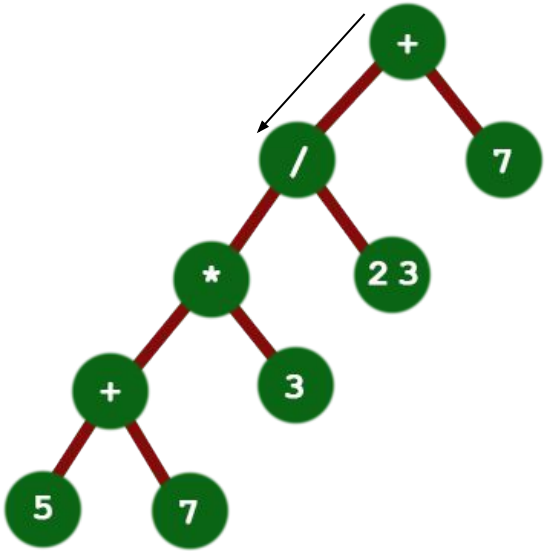

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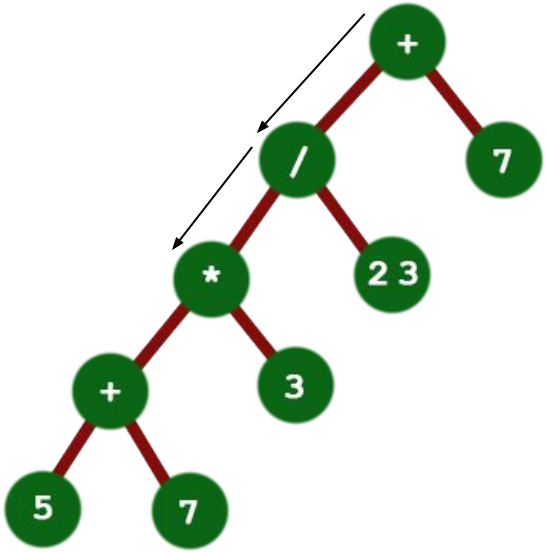

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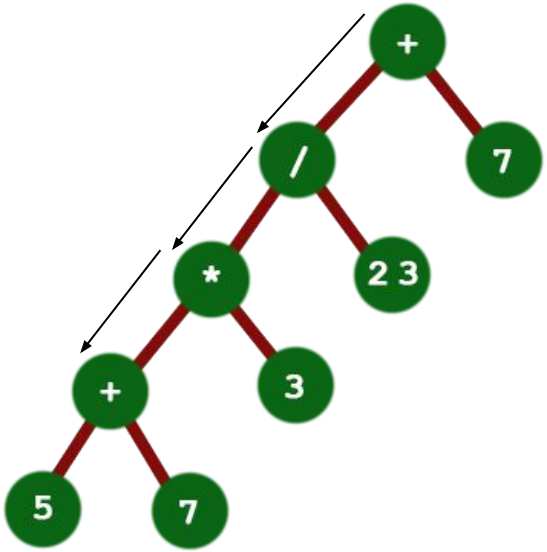

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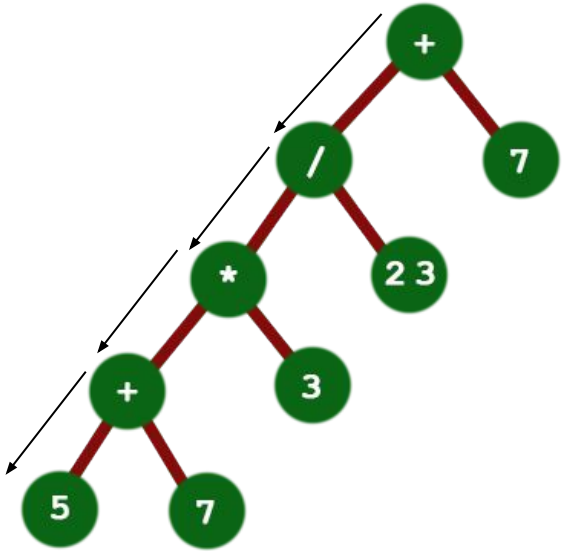

+ / * + 5

Reverse Polish Notation

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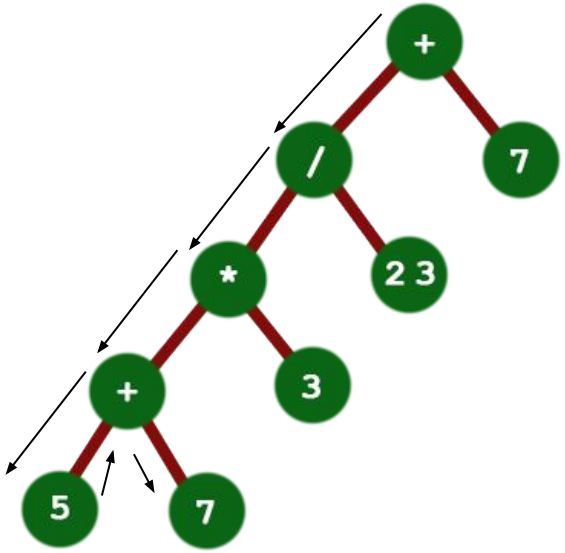

+ / * + 5 7

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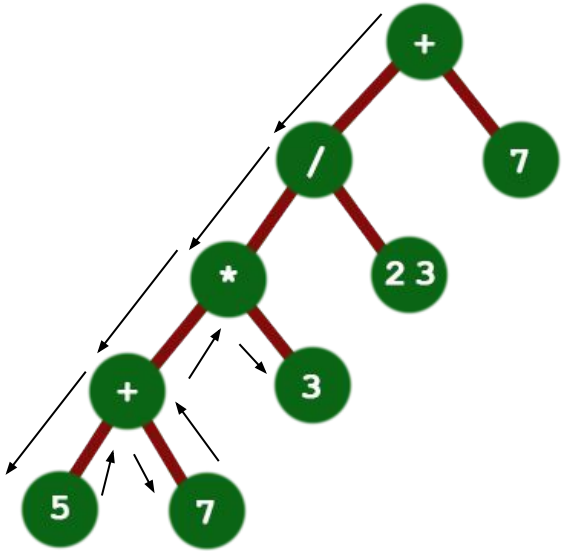

+ / * + 5 7 3

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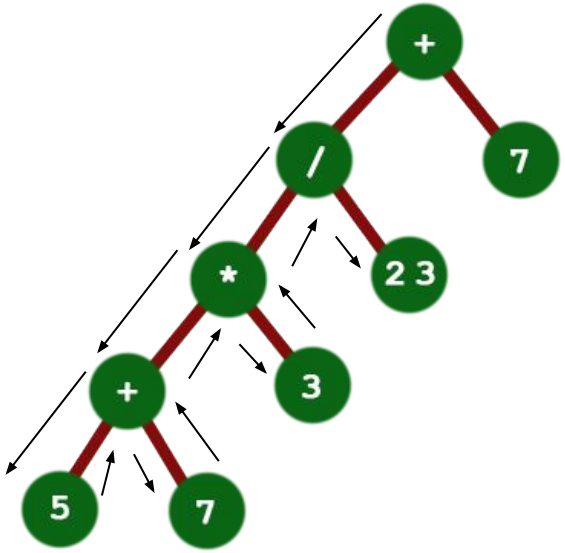

+ / * + 5 7 3 23

Reverse Polish Notation

Answer: TREES!

Operators become nodes; operands become leaves.

We can then walk the tree to produce our output. There are three methods.

Pre-order: Node, Left child, Right child

In-order: Left child, Node, Right child

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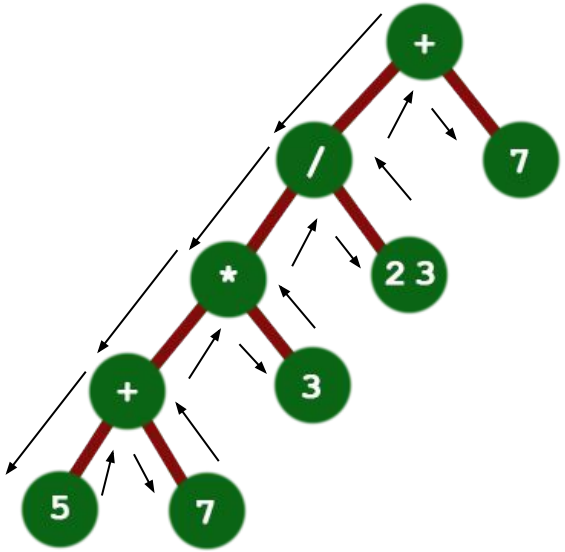

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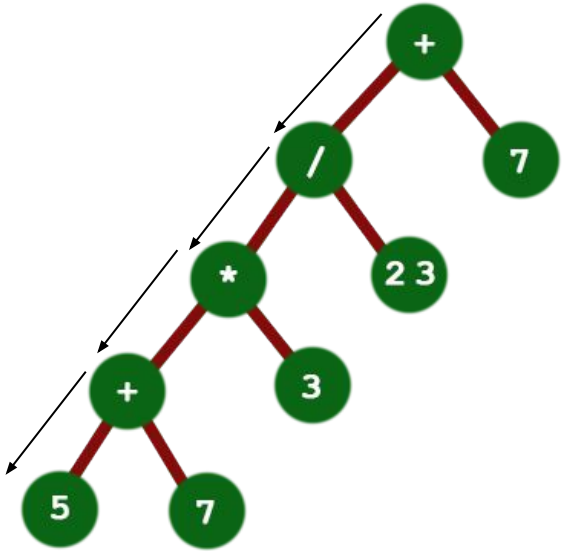

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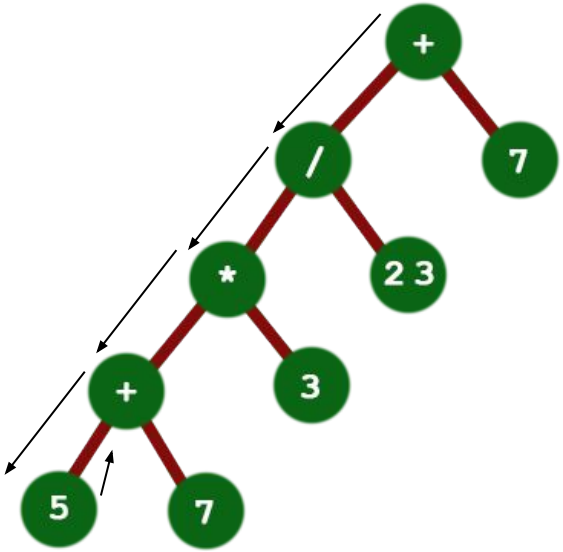

5 +

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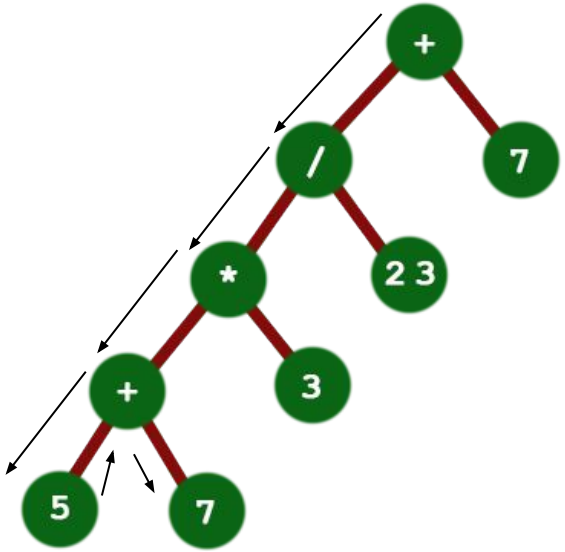

5 + 7

Reverse Polish Notation

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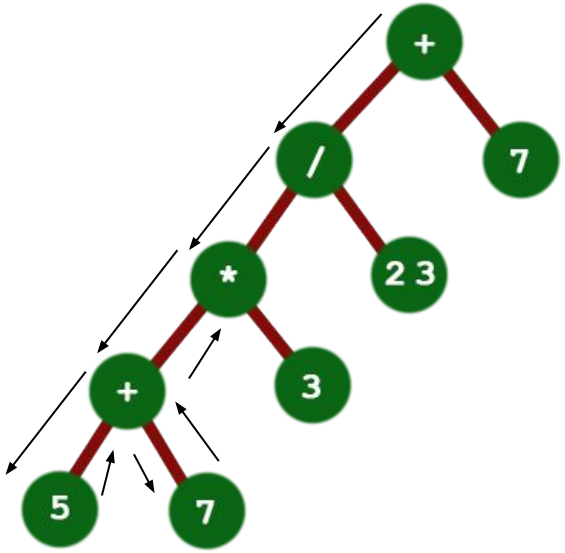

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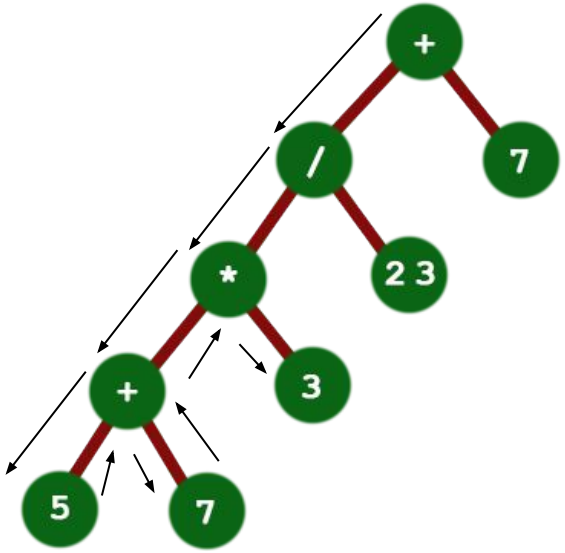

5 + 7 * 3

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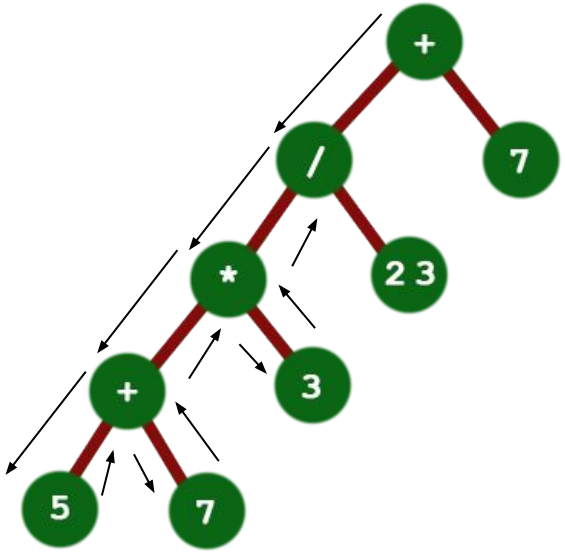

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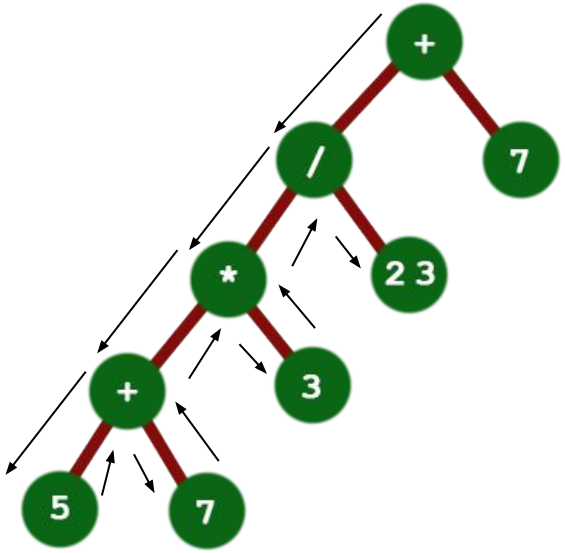

5 + 7 * 3 / 23

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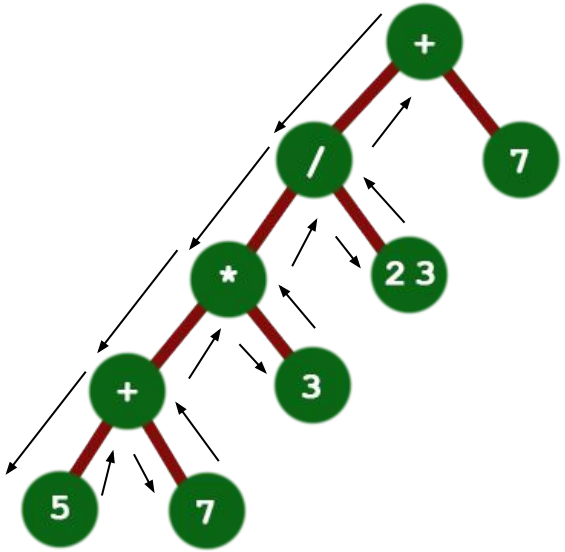

5 + 7 * 3 / 23 +

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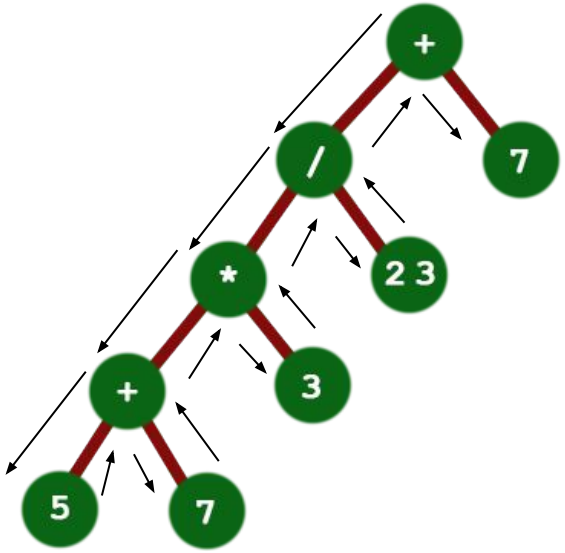

5 + 7 * 3 / 23 + 7

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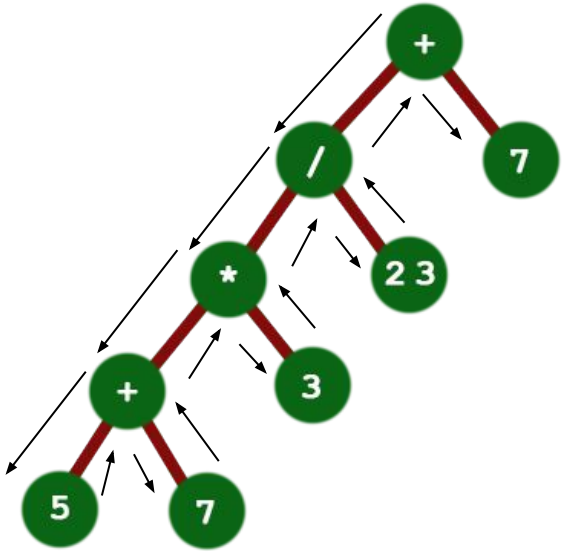

(((5 + 7) * 3) / 23) + 7

(Operator precedence matters!)

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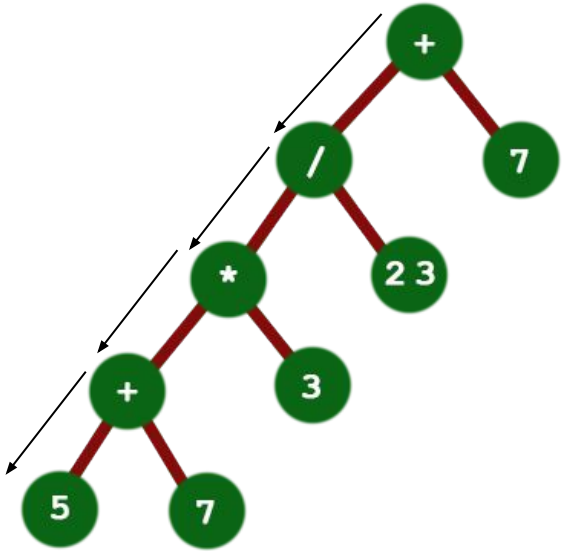

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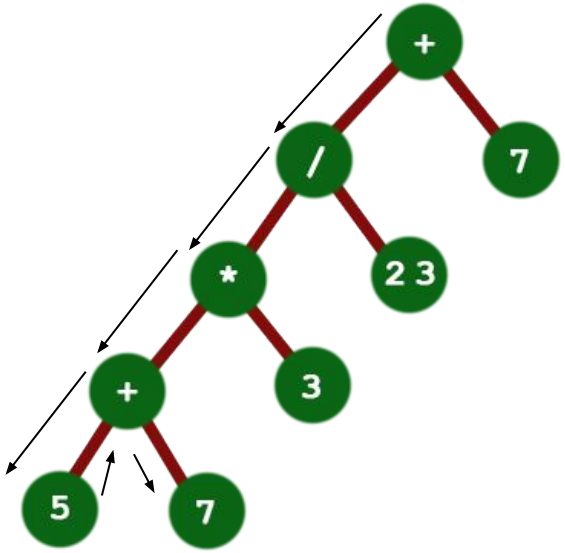

5 7

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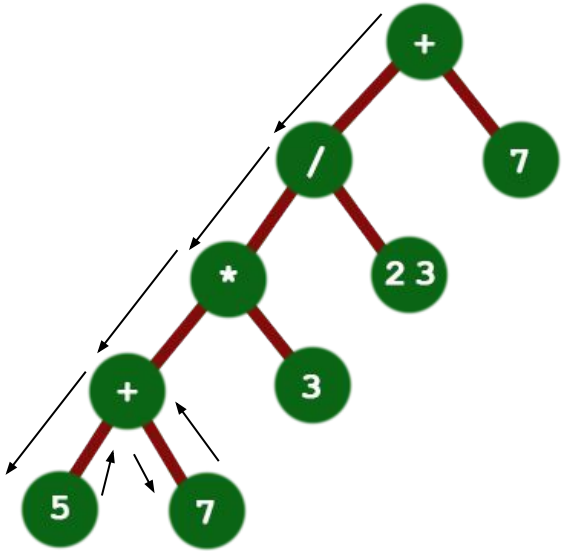

5 7 +

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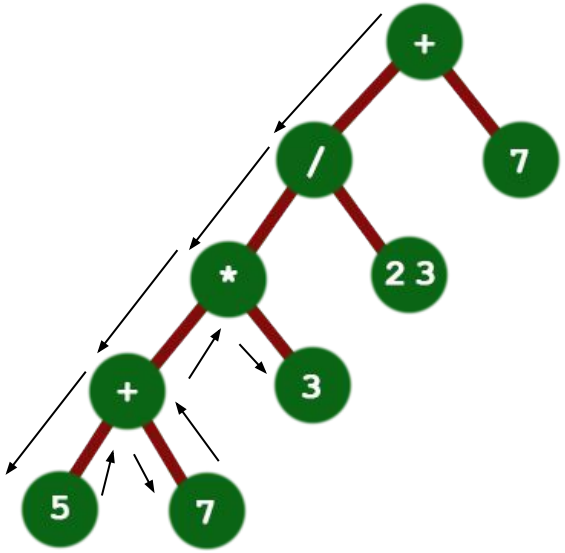

5 7 + 3

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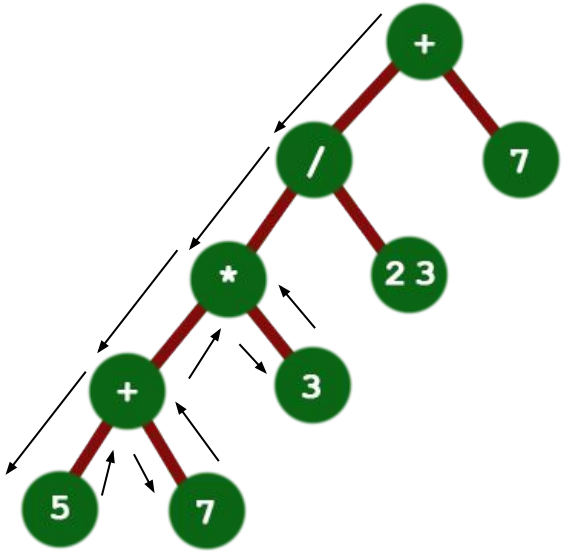

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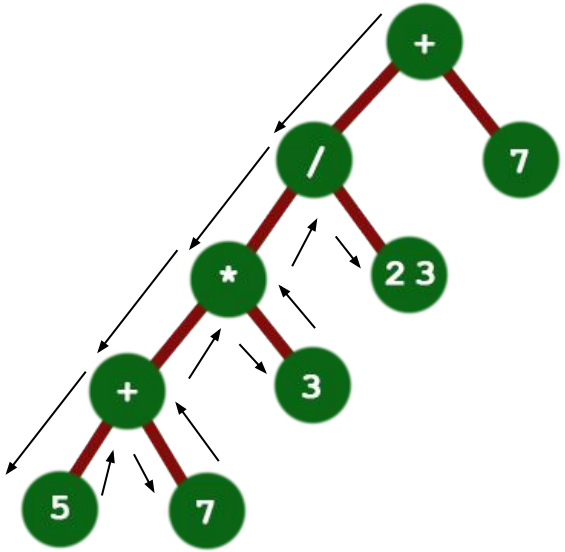

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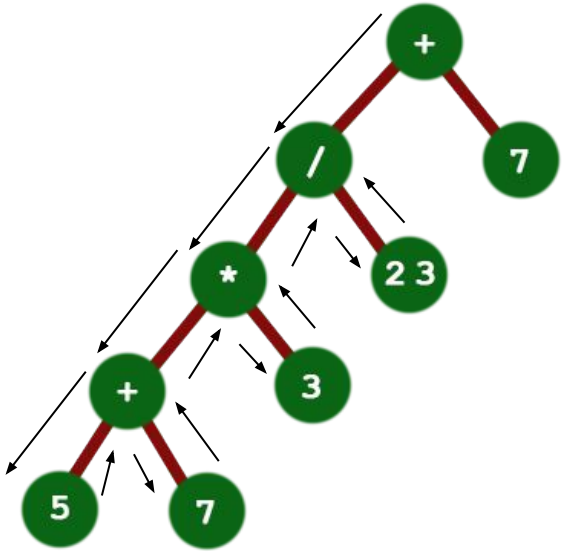

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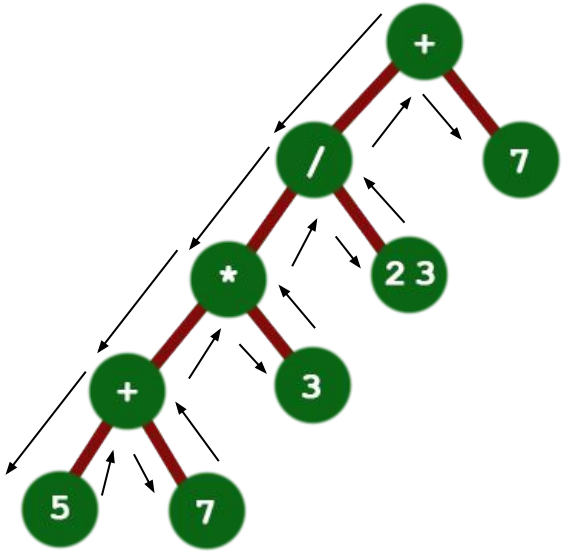

5 7 + 3 * 23 / 7

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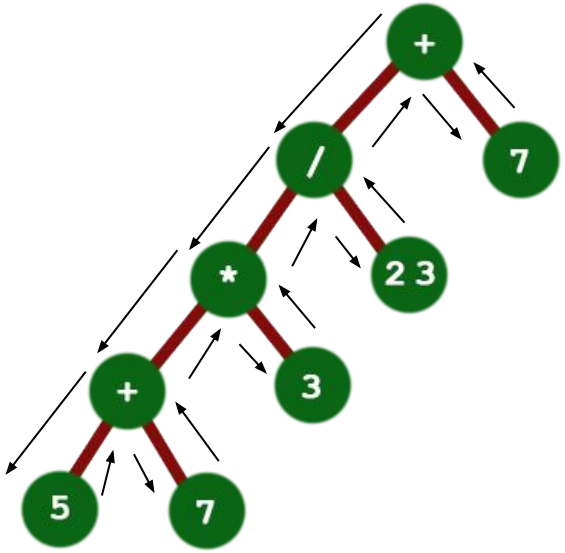
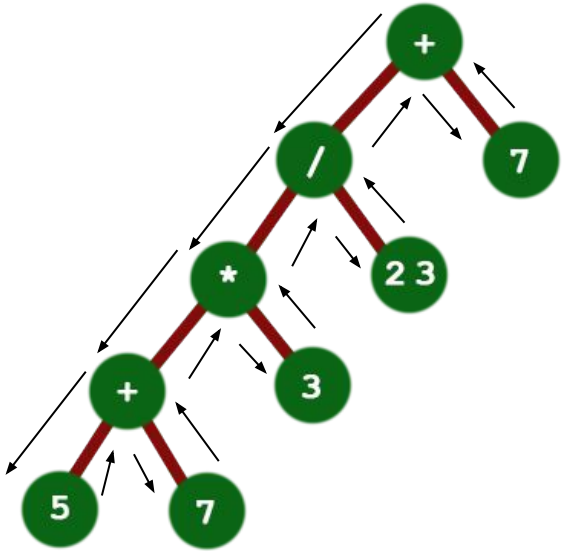


5 7 + 3 * 23 / 7 +

Reverse Polish Notation

How do we get it into the tree in the first place?



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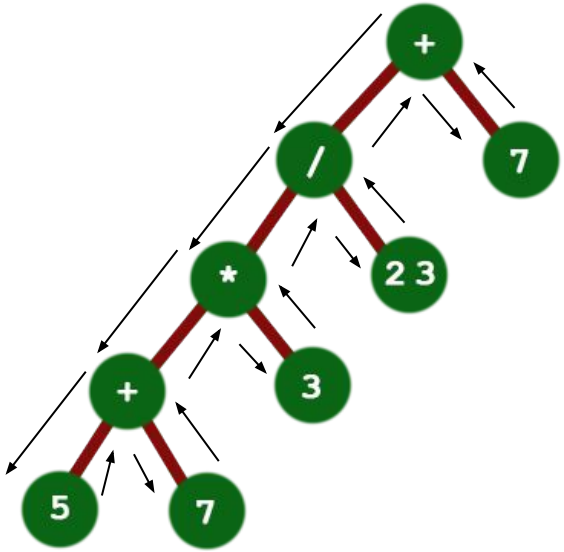
How do we get it into the tree in the first place?

Parsers will do it - creating an *Abstract Syntax Tree* - but operator precedence is messy.



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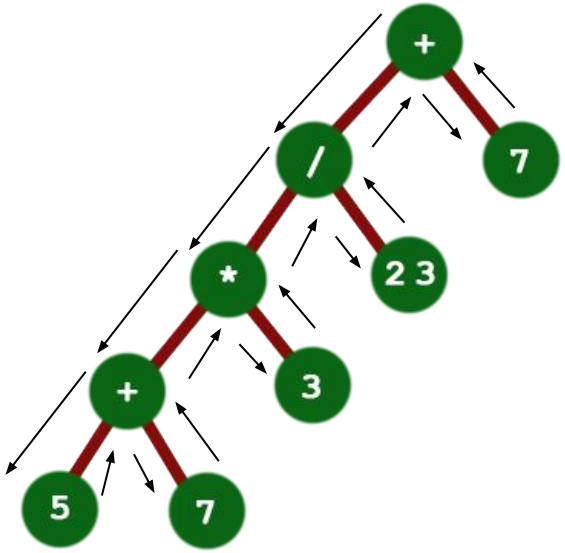
Enter Edsger Dijkstra again - his “shunting yard” algorithm is outside the scope of this talk.

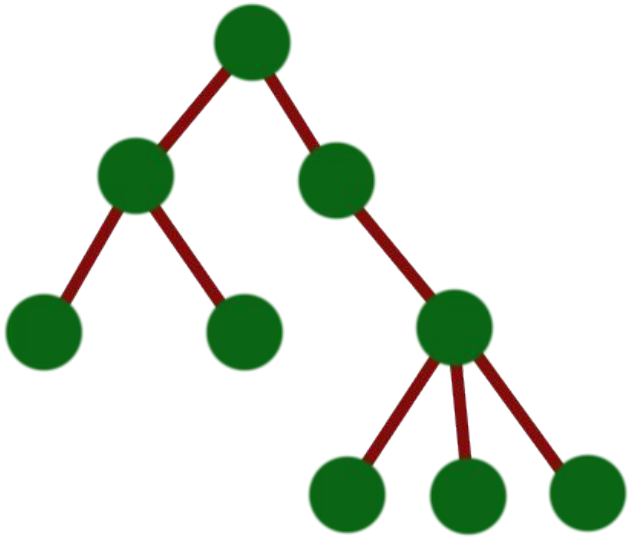


5 7 + 3 * 23 / 7 +

Nested sets

Nested sets

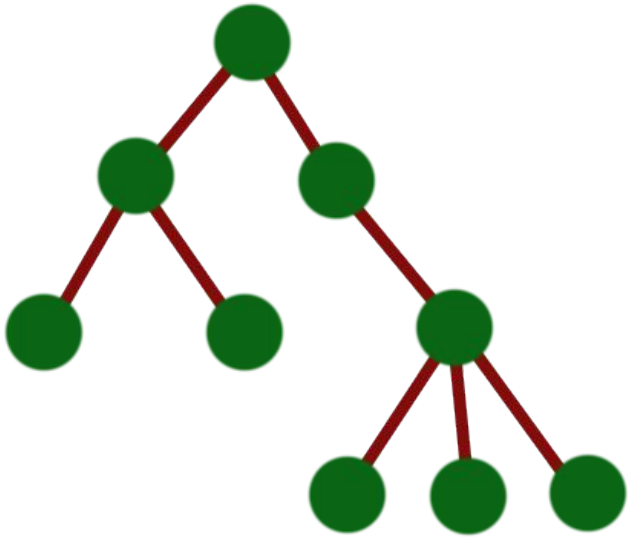
Problem: how do you store a tree in a database?



Nested sets

Problem: how do you store a tree in a database?

Answer: use a GraphDB like neo4j

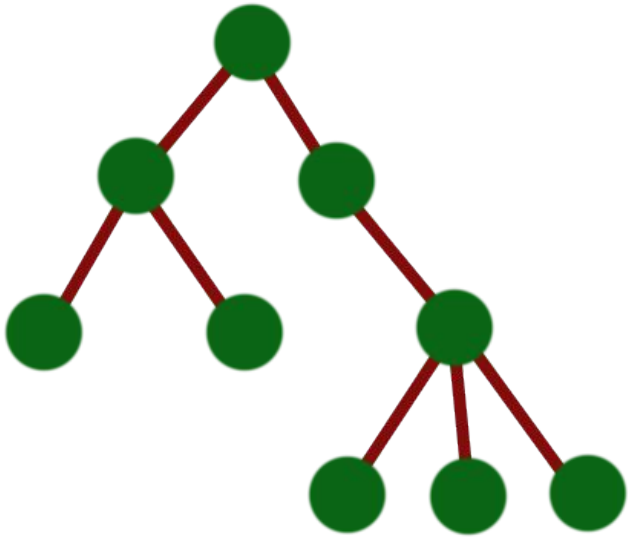


HIPSTER OVERLOAD!

(Fine, we're just afraid of change)

Nested sets

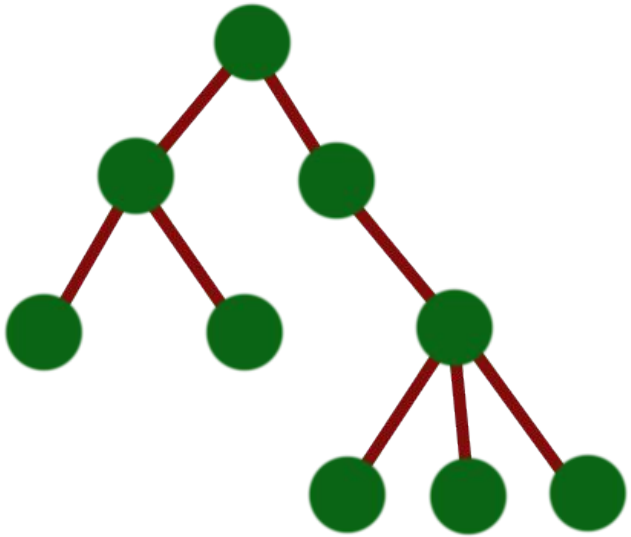
Problem: how do you store a tree in a RDBMS?



Nested sets

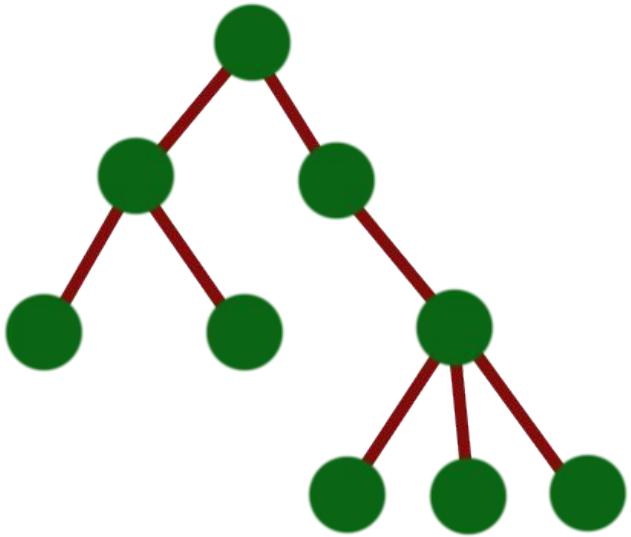
Problem: how do you store a tree in a RDBMS?

Easy! Each record has an id and a parent id; the root is the record with no parent id.



Nested sets

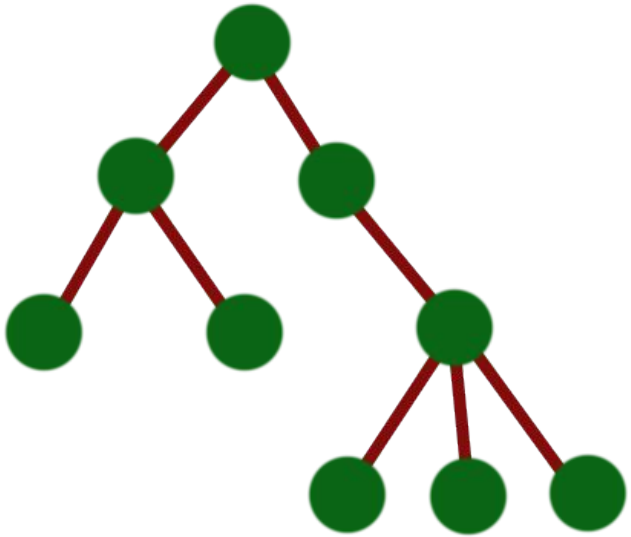
Problem: how do you retrieve all of a node’s antecedents in a RDBMS?



Nested sets

Problem: how do you retrieve all of a node's antecedents in a RDBMS?

Easy! Get that node's id, find all records with that as a parent id. Then for each of those records find all records with the parent id matching that record's id. Then for each of *those* records…



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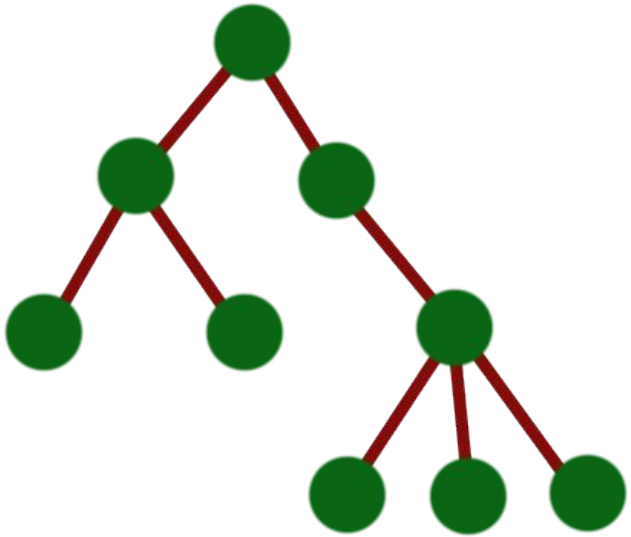
The above solution can easily spiral out of control and generate huge numbers of queries.



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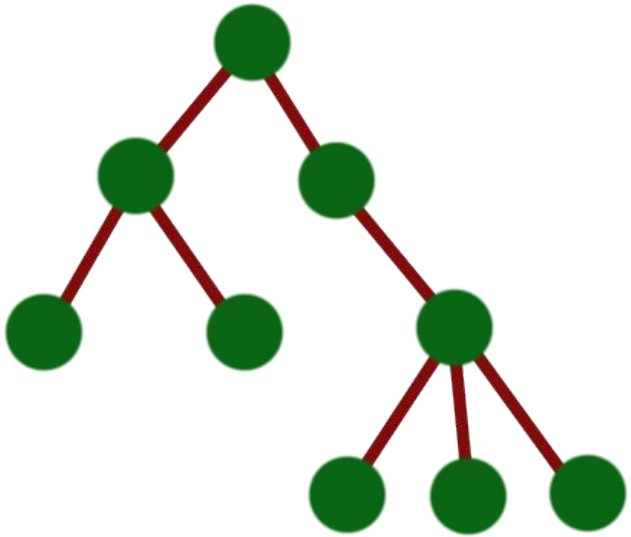

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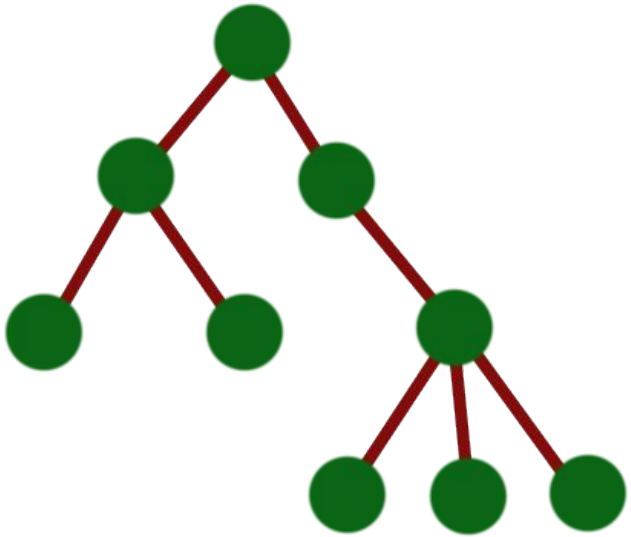
Add two more columns to your records, *left* and *right*.



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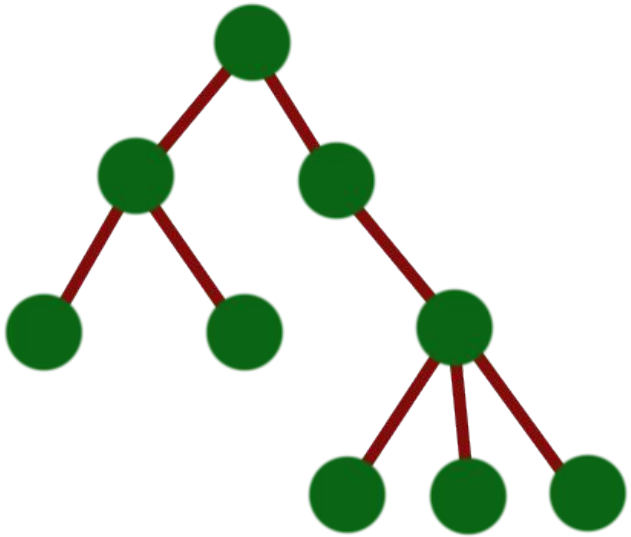
Then to store the tree, walk it while counting the times you enter a node. If it’s your first time there, set its left value to the counter; if you’ve been there before, set its right value to the counter.



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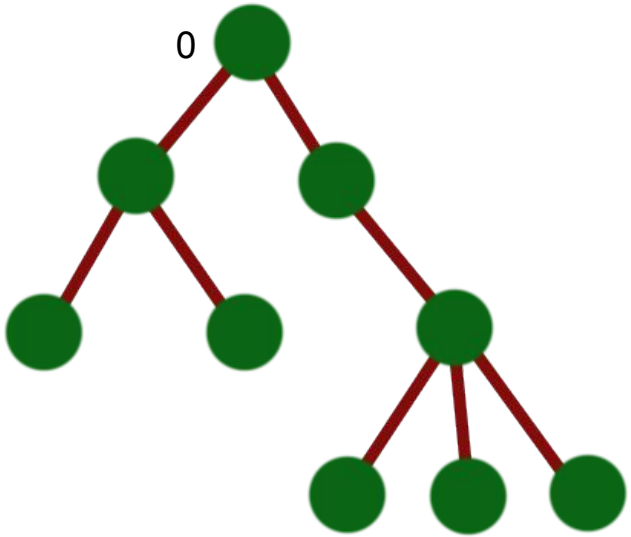

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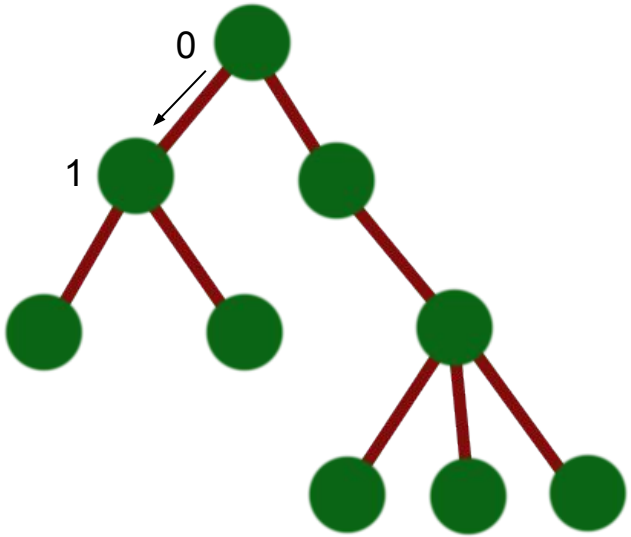

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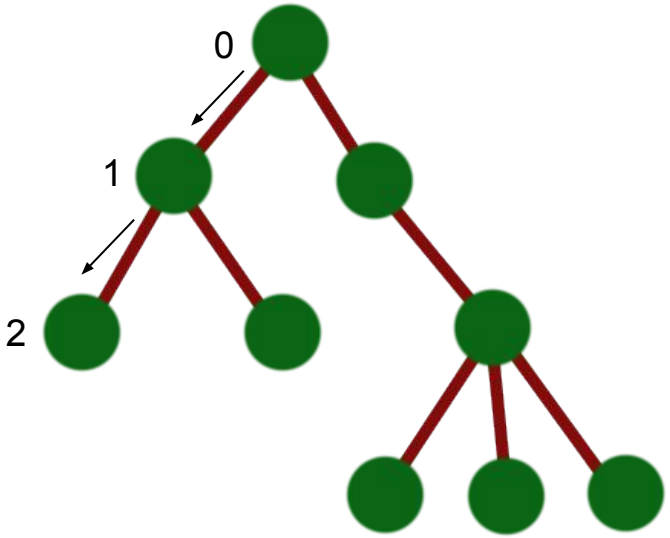

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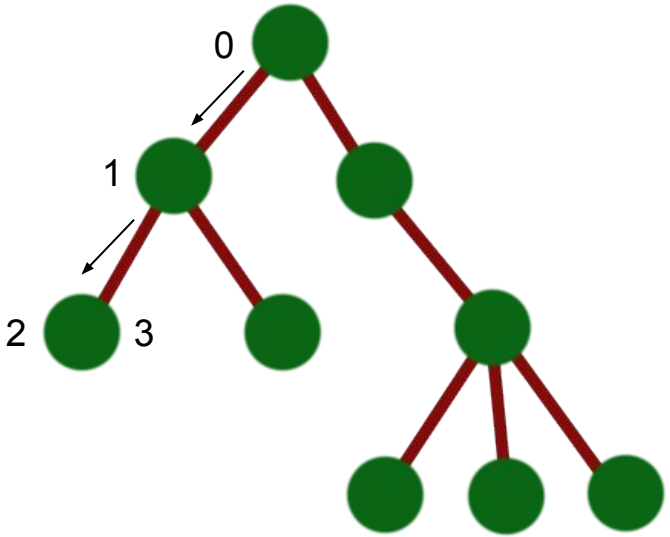
If there are no more children to walk, set the node's right value to the counter.



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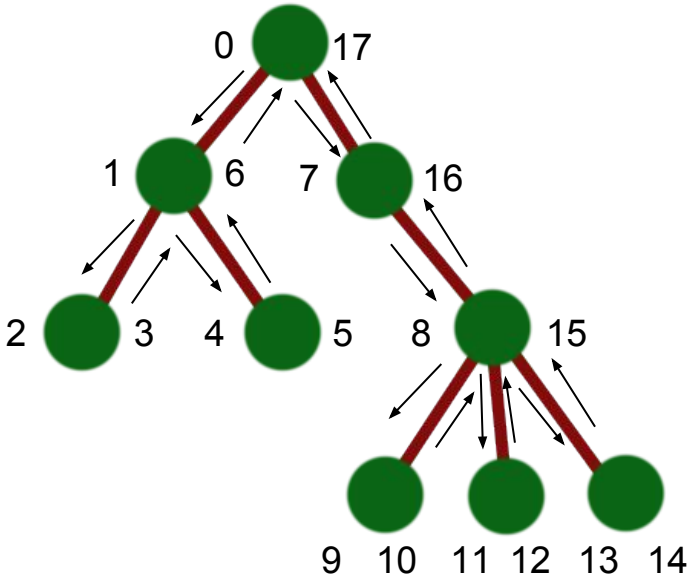
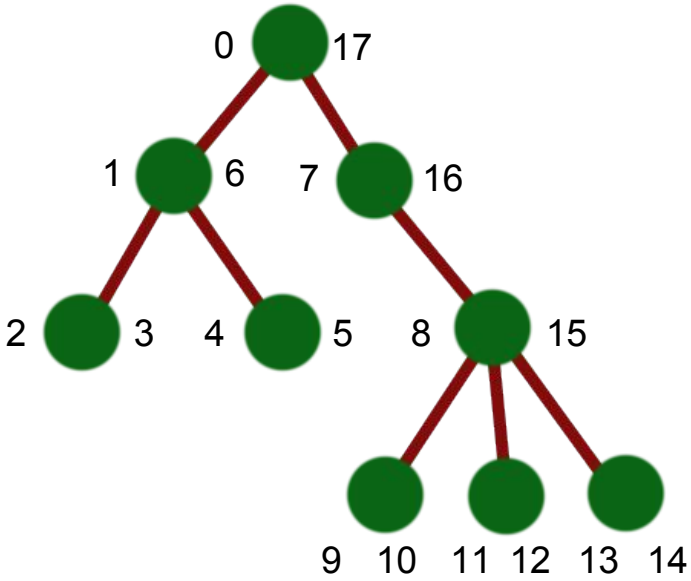


Nested sets

Every node now has the property that its children have left and right values between its own.



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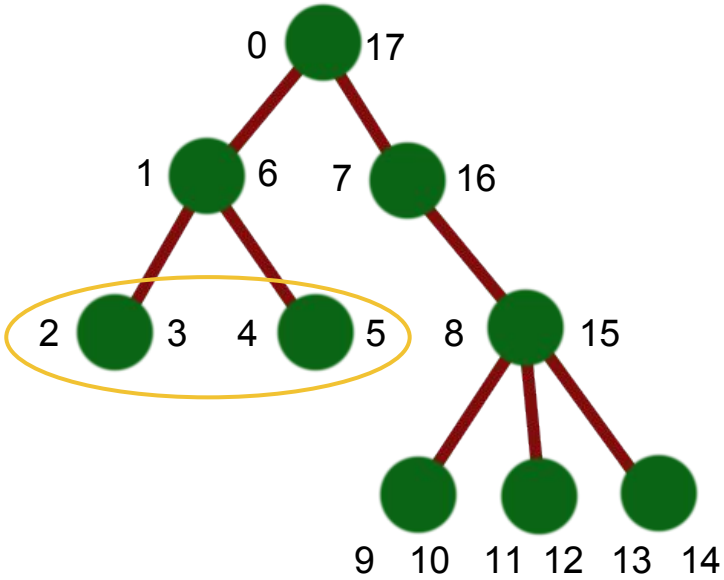
Every node now has the property that its children have left and right values between its own.

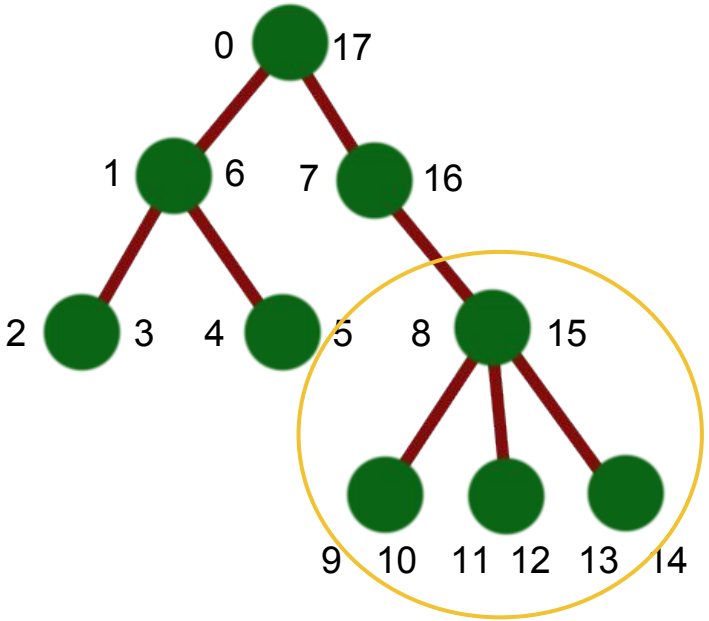
```
SELECT * FROM nodes
WHERE left > 1 AND right < 6
```



Nested sets

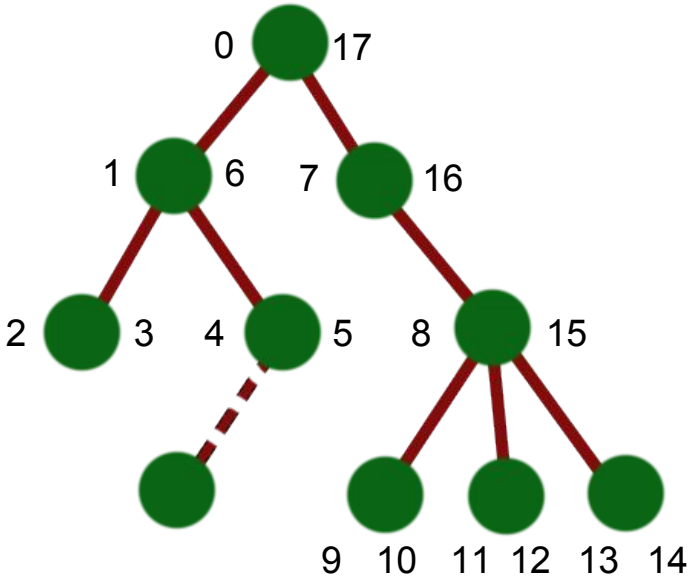
Every node now has the property that its children have left and right values between its own.

```
SELECT * FROM nodes
WHERE left > 7 AND right < 16
```



Nested sets

Insertion in SQL is then easy - with the left value for the parent node:



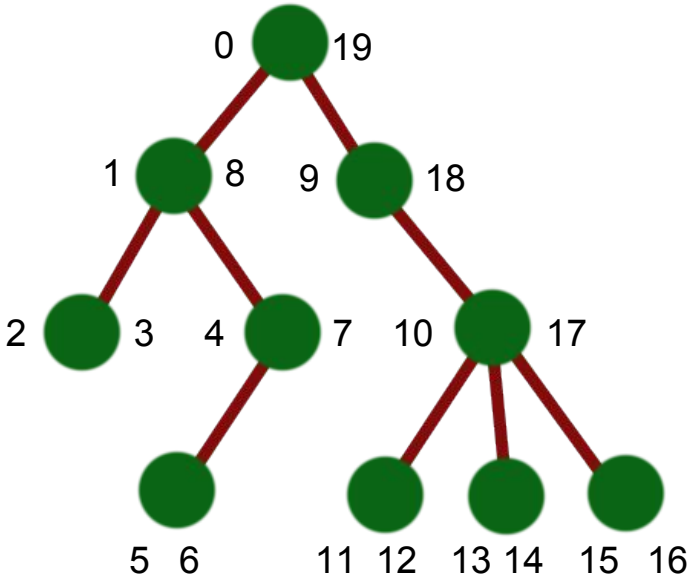
Nested sets

Insertion in SQL is then easy - with the left value for the parent node as *pleft*:

```
UPDATE node SET left = left + 2
WHERE left > pleft;

UPDATE node SET right = right + 2
WHERE right > pleft;

INSERT INTO node (left, right)
VALUES (pleft + 1, pleft + 2);
```

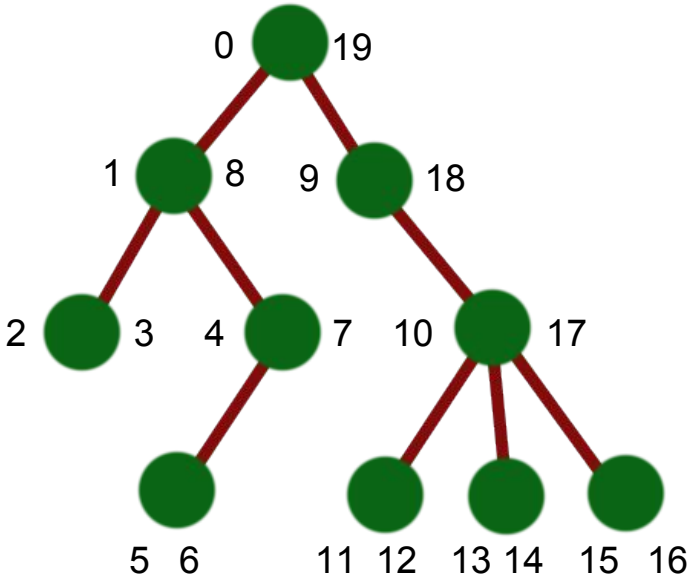


Nested sets

Deletion in SQL is easy too - with the left value for the node to be deleted as *pleft*:

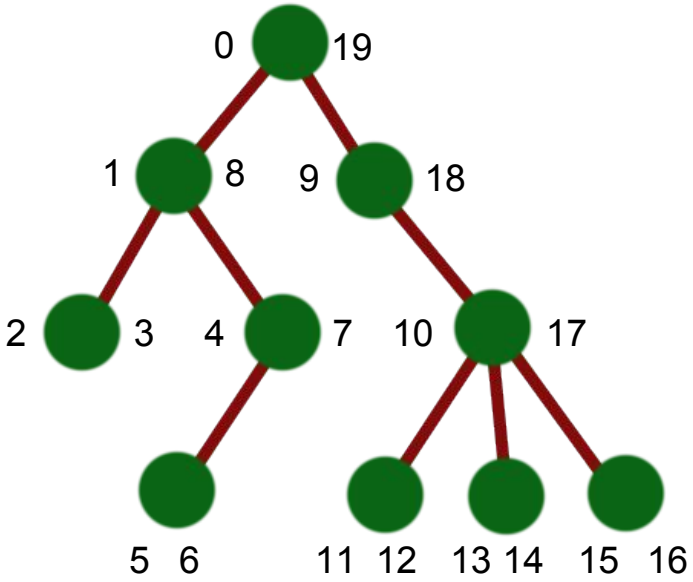
```
UPDATE node SET left = left - 2
WHERE left > pleft;

UPDATE node SET right = right - 2
WHERE right > pleft;
```



Nested sets

There are extensions to do this for you in both versions of Doctrine, Eloquent and Propel.



In Summary

Graphs are cool!

Questions?

Have you been affected by any of the issues in this talk?

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https://joind.in/talk/af2d3